

VECTORS (Q 2, PAPER 2)

2007

2 (a) $\vec{x} = -2\vec{i} + 5\vec{j}$ and $\vec{xy} = -6\vec{i} - 8\vec{j}$. Express \vec{y} in terms of \vec{i} and \vec{j} .

(b) $\vec{a} = 5\vec{i}$ and $\vec{b} = \sqrt{3}\vec{i} + 3\vec{j}$.

(i) Show that \vec{ab} is not perpendicular to \vec{b} .

(ii) Find the value of the real number k , given that $\vec{c} = k\vec{b}$ and $\vec{ac} \perp \vec{b}$.

(c) $\vec{p} = 3\vec{i} + 4\vec{j}$ and $\vec{q} = 5\vec{i} + 12\vec{j}$.

$$\vec{r} = \frac{65t}{16} \left(\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{q}}{|\vec{q}|} \right), \text{ where } t > 0.$$

(i) Express \vec{r} in terms of \vec{i} and \vec{j} .

(ii) Find $\vec{p} \cdot \vec{r}$ and $\vec{q} \cdot \vec{r}$.

(iii) Hence, show that r is on the bisector of $\angle poq$, where o is the origin.

SOLUTION

2 (a)

$$\vec{x} = -2\vec{i} + 5\vec{j}$$

$$\vec{ab} = \vec{b} - \vec{a} \dots\dots \textcircled{1}$$

$$\vec{xy} = \vec{y} - \vec{x} = -6\vec{i} - 8\vec{j} \Rightarrow \vec{y} = -6\vec{i} - 8\vec{j} + \vec{x}$$

$$\Rightarrow \vec{y} = -6\vec{i} - 8\vec{j} - 2\vec{i} + 5\vec{j} = -8\vec{i} - 3\vec{j}$$

2 (b) (i)

$$\vec{a} = 5\vec{i} \text{ and } \vec{b} = \sqrt{3}\vec{i} + 3\vec{j}.$$

If two vectors are perpendicular their dot product is zero.

$$\vec{ab} \cdot \vec{b} = (\vec{b} - \vec{a}) \cdot \vec{b} = ((\sqrt{3} - 5)\vec{i} + 3\vec{j}) \cdot (\sqrt{3}\vec{i} + 3\vec{j})$$

$$\sqrt{3}(\sqrt{3} - 5) + 9 = 3 - 5\sqrt{3} + 9 = 12 - 5\sqrt{3} \neq 0. \text{ Therefore, they are not perpendicular.}$$

2 (b) (ii)

$$\vec{c} = k\vec{b} = k(\sqrt{3}\vec{i} + 3\vec{j}) = \sqrt{3}k\vec{i} + 3k\vec{j}$$

$$\vec{ac} = \vec{c} - \vec{a} = \sqrt{3}k\vec{i} + 3k\vec{j} - 5\vec{i} = (\sqrt{3}k - 5)\vec{i} + 3k\vec{j}$$

If $\vec{ac} \perp \vec{b} \Rightarrow \vec{ac} \cdot \vec{b} = 0$.

$$\therefore \vec{ac} \cdot \vec{b} = ((\sqrt{3}k - 5)\vec{i} + 3k\vec{j}) \cdot (\sqrt{3}\vec{i} + 3\vec{j}) = 0$$

$$\Rightarrow \sqrt{3}(\sqrt{3}k - 5) + 9k = 0 \Rightarrow 3k - 5\sqrt{3} + 9k = 0$$

$$\Rightarrow 12k = 5\sqrt{3} \Rightarrow k = \frac{5\sqrt{3}}{12}$$

$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{i} = 0$ 9

$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = |\vec{i}|^2 = |\vec{j}|^2 = 1$ 10

TRICK: Just multiply the \vec{i} coefficients and multiply the \vec{j} coefficients.

2 (c) (i)

$$\vec{r} = \frac{65t}{16} \left(\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{q}}{|\vec{q}|} \right) = \frac{65t}{16} \left(\frac{3\vec{i} + 4\vec{j}}{\sqrt{9+16}} + \frac{5\vec{i} + 12\vec{j}}{\sqrt{25+144}} \right)$$

$$= \frac{65t}{16} \left(\frac{3\vec{i} + 4\vec{j}}{5} + \frac{5\vec{i} + 12\vec{j}}{13} \right) = \frac{65t}{16} \left(\frac{39\vec{i} + 52\vec{j} + 25\vec{i} + 60\vec{j}}{65} \right)$$

$$= \frac{65t}{16} \left(\frac{3\vec{i} + 4\vec{j}}{5} + \frac{5\vec{i} + 12\vec{j}}{13} \right) = \frac{65t}{16} \left(\frac{64\vec{i} + 112\vec{j}}{65} \right) = t(4\vec{i} + 7\vec{j})$$

Unit Vector: $\frac{\vec{r}}{|\vec{r}|} = \frac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ 6

2 (c) (ii)

$$\vec{p} \cdot \vec{r} = (3\vec{i} + 4\vec{j}) \cdot (4t\vec{i} + 7t\vec{j}) = 12t + 28t = 40t$$

$$\vec{q} \cdot \vec{r} = (5\vec{i} + 12\vec{j}) \cdot (4t\vec{i} + 7t\vec{j}) = 20t + 84t = 104t$$

2 (c) (iii)

$$\vec{p} \cdot \vec{r} = |\vec{p}| |\vec{r}| \cos \theta \Rightarrow \cos \theta = \frac{\vec{p} \cdot \vec{r}}{|\vec{p}| |\vec{r}|}$$

$$\vec{q} \cdot \vec{r} = |\vec{q}| |\vec{r}| \cos \theta \Rightarrow \cos \theta = \frac{\vec{q} \cdot \vec{r}}{|\vec{q}| |\vec{r}|}$$

If r bisects the angle $\Rightarrow \frac{\vec{p} \cdot \vec{r}}{|\vec{p}| |\vec{r}|} = \frac{\vec{q} \cdot \vec{r}}{|\vec{q}| |\vec{r}|}$

$$\Rightarrow \frac{\vec{p} \cdot \vec{r}}{|\vec{p}| |\vec{r}|} = \frac{40t}{5\sqrt{16t^2 + 49t^2}} = \frac{40t}{5\sqrt{65t^2}} = \frac{40t}{5t\sqrt{65}} = \frac{8}{\sqrt{65}}$$

$$\Rightarrow \frac{\vec{q} \cdot \vec{r}}{|\vec{q}| |\vec{r}|} = \frac{104t}{13t\sqrt{65}} = \frac{8}{\sqrt{65}}$$

Therefore, r is on the bisector of $\angle poq$.

Dot product: $\vec{ab} \cdot \vec{ac} = |\vec{ab}| |\vec{ac}| \cos \theta$ 8

