# DISCRETE MATHS (Q 6 & 7, PAPER 2)

## LESSON NO. 5: HARD PROBABILITY

**2006**

6 (c) There are thirty days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.

(i) What is the probabilty that all seven students have the same birthday?

(ii) What is the probability that all seven students have different birthdays?

(iii) Show that the probability that at least two have the same birthday is greater than 0·5?

**SOLUTION**

**6 (c) (i)**

*p*(A born on a day in June AND THEN B born on the same day as A AND THEN C is born on the same day as A AND THEN D is born on the same day as A AND THEN E is born on the same day as A AND THEN F is born on the same day as A AND THEN G is born on the same day as A)

$$=\frac{30}{30}\times\frac{1}{30}\times\frac{1}{30}\times\frac{1}{30}\times\frac{1}{30}\times\frac{1}{30}\times\frac{1}{30}=\frac{1}{30^6}$$

**6 (c) (ii)**

*p*(A born on a weekday AND THEN B born on a different day to A, AND THEN C born on a different day to B and A, AND THEN D born on a different day to C and B and A, AND THEN E born on a different day to D and C and B and A, AND THEN F born on a different day to E and D and C and B and A, AND THEN G born on a different day to F and E and D and C and B and A)

$$=\frac{30}{30}\times\frac{29}{30}\times\frac{28}{30}\times\frac{27}{30}\times\frac{26}{30}\times\frac{25}{30}\times\frac{24}{30}=\frac{2639}{5625}$$

**6 (c) (iii)**

At least one = 1 − *p*(None)
At least two = 1 − *p*(None or one) etc... ...... 15

*p*(At least two have the same birthday = 1 − *p*(All seven have different birthdays)

$$=1-\frac{2639}{5625}=\frac{2986}{5625}=0\cdot 53>0\cdot 5$$

**2006**

7 (b) For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as the winning combination.

(i) How many different combinations are possible?

(ii) How many of all the possible combinations will match exactly three numbers with the winning combination?

(iii) How many of all the possible combinations will match exactly three numbers with the winning combination?

(iv) Show that the probability of matching at least three numbers with the winning combination is approximately 0·014.

**SOLUTION**

**7 (b) (i)**

How many ways can you pick 5 number from 35 numbers: ${}^{35}C_5 = 324,632$

**7 (b) (ii)**

There are 5 winning (**W**) numbers and 30 non-winning (**NW**) numbers. To match 4 winning numbers, you need to pick 4 from 5 winning numbers and 1 from 30 non-winning numbers.

| 5**W** | 30**NW** |
|---|---|
| ↓ | ↓ |
| 4 Nos. | 1 No |

No. of match 4 combinations: ${}^{5}C_4 \times {}^{30}C_1 = 5 \times 30 = 150$

**7 (b) (iii)**

No. of match 3 combinations: ${}^{5}C_3 \times {}^{30}C_2 = 10 \times 435 = 4,350$

| 5**W** | 30**NW** |
|---|---|
| ↓ | ↓ |
| 3 Nos. | 2 Nos. |

**7 (b) (iii)**

*p*(Matching at least 3 Nos.) = *p*(Matching 3) + *p*(Matching 4) + *p*(Matching 5)

$$= \frac{4350}{324632} + \frac{150}{324632} + \frac{1}{324632} = \frac{4501}{324632} = 0 \cdot 014$$

**2005**

6 (c) Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(i) Find the probability that the card numbered 8 is not drawn.

(ii) Find the probability that all three cards drawn have odd numbers.

(iii) Find the probability that the sum of the numbers on the cards drawn is greater than the sum of the numbers on the cards not drawn.

**SOLUTION**

**6 (c)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|

**6 (c) (i)**

Find the probability of a particular combination and then calculate the number of ways in which that combination can take place.

$p(\text{Not } 8, \text{Not } 8, \text{Not } 8) = \frac{8}{9} \times \frac{7}{8} \times \frac{6}{7} = \frac{6}{9} = \frac{2}{3}$

| N8 | N8 | N8 |
|---|---|---|

**6 (c) (ii)**

$p(\text{Odd}, \text{Odd}, \text{Odd}) = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

| Odd | Odd | Odd |
|---|---|---|

**6 (c) (iii)**

Write out all the possibilities:

| | | |
|---|---|---|
| 7 | 8 | 9 |
| 6 | 8 | 9 |
| 5 | 8 | 9 |
| 4 | 8 | 9 |

What is the probability of picking the first combination?

$p(7, 8, 9) = \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7} \times 3 = \frac{1}{168}$

There are four such possibilities.

$p$(Sum of cards drawn is greater than those not drawn)

$= \frac{1}{168} \times 4 = \frac{1}{42}$

**2005**

7 (b) There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(i) the four discs are blue

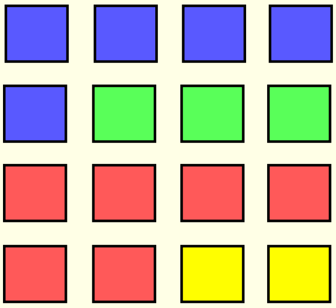
(ii) the four discs are the same colour

(iii) all four discs are different in colour

(iv) two of the discs are blue and two are not blue?

**Solution**

**7 (b)**



**7 (b) (i)**

$p(\text{B, B, B, B}) = \frac{5}{16} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{1}{364}$

**7 (b) (ii)**

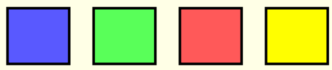
$p(\text{All the same colour}) = p(\text{B, B, B, B}) + p(\text{R, R, R, R})$

$= \frac{1}{364} + \frac{6}{16} \times \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} = \frac{1}{364} + \frac{3}{364} = \frac{4}{364} = \frac{1}{91}$

**7 (b) (iii)**

To find the probability that the four discs drawn are different in colour, find out the probability of picking four definite colours in a definite order and then multiply this probability by the number of combinations of the four colours.

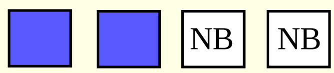
$p(\text{B, G, R, Y}) = \frac{5}{16} \times \frac{3}{15} \times \frac{6}{14} \times \frac{2}{13} \times 24 = \frac{9}{91}$



No. of combinations = 4! = 24

**7 (b) (iii)**

$p(\text{B, B, Not Blue (NB), NB}) = \frac{5}{16} \times \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13} \times 6 = \frac{55}{182}$




No. of combinations $= \frac{4!}{2!2!} = 6$

**2004**

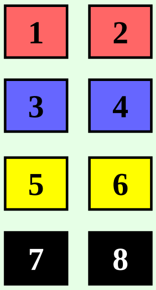
6 (c) Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.
Four cards are selected at random from the eight cards.
Find the probability that the four cards selected are:

(i) all of different colours

(ii) two odd-numbered cards and two even-numbered cards

(iii) all of different colours, two odd-numbered and two even-numbered.

**SOLUTION**




**6 (c) (i)**

Find the probability of picking four different colours in a definite order and multiply this probability by the number of combinations of four colours.

$p(\text{Red, Blue, Yellow, Black}) = \frac{2}{8}\times\frac{2}{7}\times\frac{2}{6}\times\frac{2}{5}=\frac{1}{105}$

$p(\text{Four different colours}) = \frac{1}{105}\times 4! = \frac{8}{35}$

**6 (c) (ii)**

$p(\text{Odd, Odd, Even, Even}) = \frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{3}{5}=\frac{3}{35}$

$p(\text{Two odd-numbers and two even-numbers}) = \frac{3}{35}\times 6 = \frac{18}{35}$

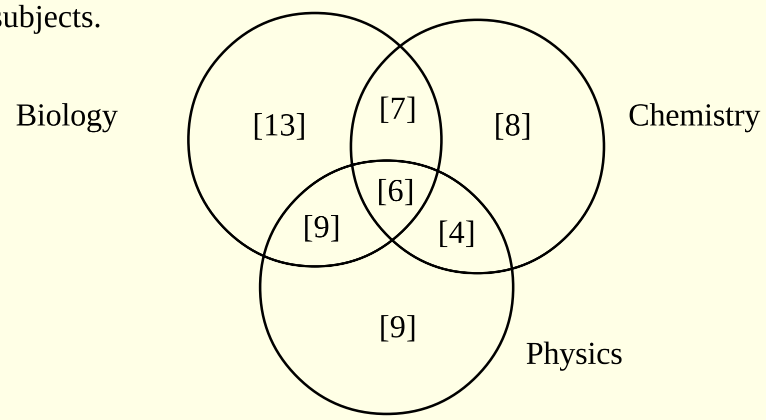
**6 (c) (iii)**

$p(\text{Odd red, Odd blue, Even yellow, Even black}) = \frac{1}{8}\times\frac{1}{7}\times\frac{1}{6}\times\frac{1}{5}=\frac{1}{1680}$

p(Four different colours, two odd-numbers and two even-numbers) $= \frac{1}{1680}\times 4!\times 6 = \frac{3}{35}$

**2004**

7 (b) In a class of 56 students, each studies at least one of the subjects Biology, Chemistry, Physics. The Venn diagram shows the number of students studying the various combinations of subjects.




(i) A student is picked at random from the whole class. Find the probability that the student does not study Biology.

(ii) A student is picked at random from those who study at least two of the subjects. Find the probability that the student does not study Biology.

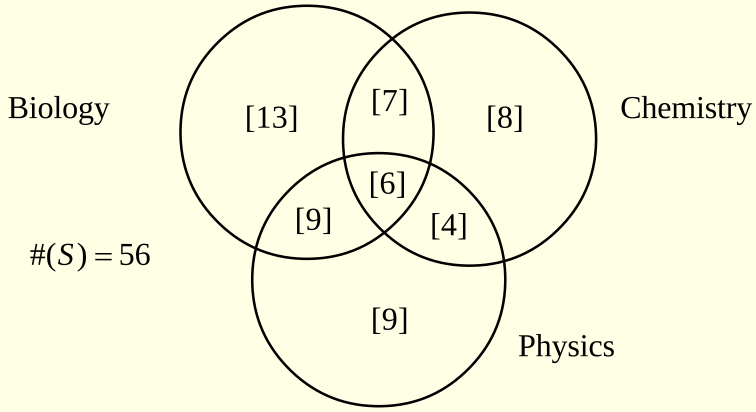
(iii) Two students are picked at random from the whole class. Find the probability that they both study Physics.

(iv) Two students are picked at random from those who study Chemistry. Find the probability that exactly one of them studies Biology.

**SOLUTION**

**7 (b)**

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)} \quad \text{...... } \textbf{12}$$




**7 (b) (i)**

No. of students not studying Biology = 8 + 4 + 9 = 21

$p$(Student does not study Biology) $= \frac{21}{56} = \frac{3}{8}$

**7 (b) (ii)**

No. of students studying at least two subjects = 7 + 9 + 4 + 6 = 26

No. of students not studying Biology from these students = 4

$p$(Student does not study Biology from those studying at least two subjects) $= \frac{4}{26} = \frac{2}{13}$

**Cont...**

**7 (b) (iii)**
No. of students studying Physics = 9 + 9 + 6 + 4 = 28

$p$(Two students studying Physics) $= \frac{28}{56} \times \frac{27}{55} = \frac{27}{110}$

**7 (b) (iv)**
No. of students studying Chemistry = 8 + 7 + 4 + 6 = 25
No. of these students studying Biology = 7 + 6 = 13
No. of these students not studying Biology = 8 + 4 = 12

$p$(One student studies Biology and one does not study Biology) $= \frac{13}{25} \times \frac{12}{24} \times 2 = \frac{13}{25}$

**2003**

6 (c) Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.

(i) What is the probability that the disc with the number 7 is drawn?

(ii) What is the probability that the three numbers on the discs drawn are odd?

(iii) What is the probability that the product of the three numbers on the discs drawn is even?

(iv) What is the probability that the smallest number on the discs drawn is 4?

**SOLUTION**

**6 (c) (i)**
Find the probability of a particular combination and then calculate the number of ways in which that combination can take place.

$p$(7, Not 7, Not 7) $= \frac{1}{10} \times \frac{9}{9} \times \frac{8}{8} = \frac{1}{10}$

| 7 | N7 | N7 |
|---|---|---|

No. of combinations $= \frac{3!}{2!} = 3$

$p$(7 is drawn) $= \frac{1}{10} \times 3 = \frac{3}{10}$

**6 (c) (ii)**

p(Odd, Odd, Odd) $= \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$

**6 (c) (iii)**
The product of three odd numbers is odd. Every other combination of three numbers is even.

$p$(Product is even) = 1 – $p$(Product is odd) $= 1 - \frac{1}{12} = \frac{11}{12}$

**6 (c) (iv)**

$p$(4, >4, >4) $= \frac{1}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{1}{24}$

| 4 | >4 | >4 |
|---|---|---|

No. of combinations $= \frac{3!}{2!} = 3$

$p$(Smallest number drawn is 4) $= \frac{1}{24} \times 3 = \frac{1}{8}$

**2001**

6 (c) A box contains four silver coins, two gold coins and $x$ copper coins. Two coins are picked at random, and without replacement, from the box.

(i) Write down an expression in $x$ for the probability that the two coins are copper.

If it is known that the probability of picking two copper coins is $\frac{4}{13}$,

(ii) how many coins are in the box and

(iii) what is the probability that neither of the two coins picked is copper?

**SOLUTION**

**6 (c)**

| |
|---|
| 4 Silver coins |
| 2 Gold coins |
| $x$ Copper coins |

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$ ...... 12

**6 (c) (i)**

$$p(\text{Copper, Copper}) = \frac{x}{(x+6)} \times \frac{(x-1)}{(x+5)}$$

**6 (c) (ii)**

$$\frac{x}{(x+6)} \times \frac{(x-1)}{(x+5)} = \frac{4}{13} \Rightarrow 13x(x-1) = 4(x+6)(x+5)$$

$$\Rightarrow 13x^2 - 13x = 4x^2 + 44x + 120 \Rightarrow 9x^2 - 57x - 120 = 0$$

$$\Rightarrow 3x^2 - 19x - 40 = 0 \Rightarrow (3x+5)(x-8) = 0$$

$$\Rightarrow x = -\tfrac{5}{3}, 8$$

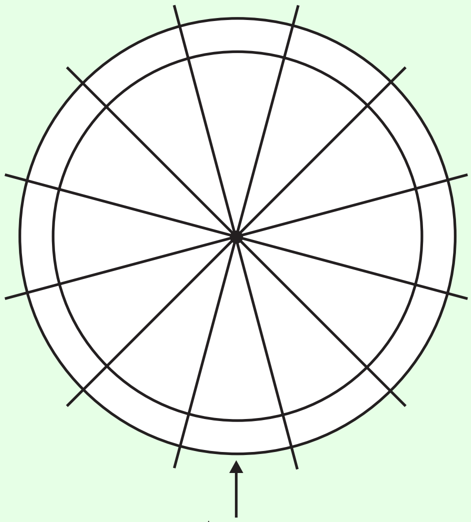
There are 8 copper coins (this number has to be a natural number.)
Total number of coins = 14

**6 (c) (iii)**

$$p(\text{Not Copper, Not Copper}) = \tfrac{6}{14} \times \tfrac{5}{13} = \tfrac{15}{91}$$

**2001**

7 (b) To play a game a player spins a wheel.
The wheel is fixed to a wall. It spins freely around its centre point. Its rim is divided equally into twelve regions. Three of the regions are coloured red. Four are coloured blue. Five are coloured green.
When the wheel stops an arrow fixed to the wall points to one of the regions. All the regions are equally likely to stop at the arrow. The colour of this region is the outcome of the game.
When the game is played twice, calculate the probability that




(i) both outcomes are green
(ii) both outcomes are the same colour
(iii) the first outcome is red and the second is green
(iv) one outcome is green and the other is blue.

**SOLUTION**

**7 (b)**

| |
|---|
| 3 Red<br>4 Blue<br>5 Green |

This problem could also be viewed as a container with 12 discs where the discs are picked with replacement.

**7 (b) (i)**

$p(\text{Green and Green}) = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$

**7 (b) (ii)**

$p(\text{Green and Green})$ OR $p(\text{Red and Red})$ OR $p(\text{Blue and Blue})$

$= \frac{5}{12} \times \frac{5}{12} + \frac{3}{12} \times \frac{3}{12} + \frac{4}{12} \times \frac{4}{12} = \frac{25}{144} + \frac{9}{144} + \frac{16}{144} = \frac{50}{144} = \frac{25}{72}$

**7 (b) (iii)**

$p(\text{Red AND THEN Green}) = \frac{3}{12} \times \frac{5}{12} = \frac{15}{144} = \frac{5}{48}$

**7 (b) (iv)**

$p(\text{Green and Blue}) = \frac{5}{12} \times \frac{4}{12} \times 2 = \frac{5}{18}$