DISCRETE MATHS (Q 6 & 7, PAPER 2)

1998

6 (a) In how many ways can the letters of the word IRELAND be arranged if each letter is used exactly once in each arrangement?
In how many of these arrangements do the three vowels come together?

(b) Solve the difference equation
$$4u_{n+2} - 25u_{n+1} - 29u_n = 0, \text{ where } n \geq 0,$$
given that $u_0 = 0$ and $u_1 = 16 \cdot 5$.

(c) On an unbiased die, the numbers 1, 3 and 4 are coloured red and the numbers 2, 5 and 6 are coloured black.
(i) The die is thrown once. Find the probability of getting an even number or a red number.

(ii) The die is thrown three times with the following outcome:
the second throw shows a red number and the sum of the numbers on the first and second throws is equal to the number on the third throw.
Find the probability of this outcome.

7 (a) If p is the mean of the numbers a, b, c, d express in terms of p and k the mean of the numbers $2a + k$, $2b + k$, $2c + k$, $2d + k$.

(b) A classroom contains 15 desks which are arranged in rows.
The front row contains 3 desks.
15 students are seated at random in the classroom, 8 of whom are boys and 7 of whom are girls.
Each desk seats only one student.
What is the probability that
(i) three girls occupy the front row of desks?

(ii) there are more boys than girls seated in the front row?

(iii) there are two girls and one boy seated in the front row with the two girls seated next to each other?

(c) The numbers p, q, r have a mean $\bar{x}$ and a standard deviation σ.
(i) Express $\bar{x}$ in terms of p, q and r.

(ii) Show that
$$\sigma^2 = \tfrac{1}{3}(p^2 + q^2 + r^2) - (\bar{x})^2.$$

ANSWERS

6 (a) 5040, 720

6 (b) $u_n = 2(\frac{29}{4})^n - 2(-1)^n$

6 (c) (i) $\frac{5}{6}$ (ii) $\frac{5}{108}$

7 (a) $2p + k$

7 (b) (i) $\frac{1}{13}$ (ii) $\frac{36}{65}$ (iii) $\frac{16}{65}$

7 (c) (i) $\frac{1}{3}(p+q+r)$