LINE (Q 3, PAPER 2)

2007

3 (a) Find the area of the triangle with vertices $(1, 1)$, $(8, -5)$ and $(5, -2)$.

(b) f is the transformation $(x, y) \to (x', y')$, where $x' = 4x + 2y$ and $y' = -3x - y$.
K is the line $x + y = 0$.
(i) Show that K is its own image under f.

(ii) $p(1, -1)$ and $q(3, -3)$ are two points.

Find the ratio $|pq| : |f(p)f(q)|$, giving your answer in its simplest form.

(c) Consider the equation $k(3x - 5y + 6) + l(5x - 7y + 4) = 0$, where $k, l \in \mathbf{R}$.
(i) Show that for all k and l, the given equation represents a line passing through the point of intersection of $3x - 5y + 6 = 0$ and $5x - 7y + 4 = 0$.

(ii) Find the relationship between k and l for which the given equation represents a line of slope 2.

(iii) If $k = 1$, what line through the point of intersection cannot be represented by the given equation? Justify your answer.

SOLUTION

3 (a)

$$(1,1) \to (0,0)$$
$$(8,-5) \to (7,-6)$$
$$(5,-2) \to (4,-3)$$

$$A = \tfrac{1}{2}|(7)(-3) - (-6)(4)| = \tfrac{1}{2}|-21 + 24| = \tfrac{3}{2}$$

$A = \frac{1}{2}|x_1 y_2 - x_2 y_1|$ 4

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

3 (b)

$$x' = 4x + 2y \quad \Leftarrow x' = 4x + 2y \Rightarrow \quad 3x' = 12x + 6y$$
$$2y' = -6x - 2y \quad \Leftarrow y' = -3x - y \Rightarrow \quad 4y' = -12x - 4y$$
$$\frac{-x' - 2y'}{2} = x \qquad\qquad \frac{3x' + 4y'}{2} = y$$

3 (b) (i)

K: $x + y = 0$

$$K': \frac{-x' - 2y'}{2} + \frac{3x' + 4y'}{2} = 0 \Rightarrow -x' - 2y' + 3x' + 4y' = 0 \Rightarrow 2x' + 2y' = 0$$

$$\Rightarrow x' + y' = 0$$

Therefore, K is its own image under f.

3 (b) (ii)

$p(1,-1) \Rightarrow f(p) = (2,-2)$

$q(3,-3) \Rightarrow f(q) = (6,-6)$

$|pq| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

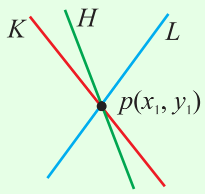
$|f(p)f(q)| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\Rightarrow |pq| : |f(p)f(q)| = 2\sqrt{2} : 4\sqrt{2} = 1:2$

3 (c)

$k(3x-5y+6)+l(5x-7y+4)=0 \Rightarrow (3k+5l)x+(-5k-7l)y+(6k+4l)=0$ ....Eqn. **1**

Concurrent Line Formula: Given any two concurrent lines, K and L, any other line, H, concurrent with these can be expressed as: $H = \mu K + \lambda L$ where $\mu,\ \lambda \in \mathbf{R}$.



3 (c) (i)

Find the point of intersection of $3x-5y+6=0$ and $5x-7y+4=0$.

$$3x-5y+6=0\,(\times 5) \rightarrow 15x-25y+30=0$$
$$5x-7y+4=0\,(\times -3) \rightarrow -15x+21y-12=0$$
$$-4y+18=0 \Rightarrow y=\tfrac{9}{2}$$

$\therefore 3x-5(\tfrac{9}{2})+6=0 \Rightarrow 3x=\tfrac{33}{2} \Rightarrow x=\tfrac{11}{2}$

Point of intersection: $(\tfrac{11}{2},\tfrac{9}{2})$

Substitute this point into Eqn. **1**:

$\therefore (3k+5l)(\tfrac{11}{2})+(-5k-7l)(\tfrac{9}{2})+(6k+4l)=\tfrac{33}{2}k+\tfrac{55}{2}l-\tfrac{45}{2}k-\tfrac{63}{2}l+6k+4l=0$

Therefore, $k(3x-5y+6)+l(5x-7y+4)=0$, where $k,\ l \in \mathbf{R}$, represents the equation of a line passing through $(\tfrac{11}{2},\tfrac{9}{2})$ for all values of k and l.

3 (c) (ii)

Slope of Eqn. **1**, $m=-\dfrac{3k+5l}{-5k-7l}=\dfrac{3k+5l}{5k+7l}=2 \Rightarrow 3k+5l=10k+14l$

$\Rightarrow 7k+9l=0$

3 (c) (iii)

The line $5x-7y+4=0$ is generated from Eqn. **1** for values of k = 0 and any value of l. If k = 1, then this line which contains the point of intersection cannot be produced from Eqn. **1**.