Integration (Q 8, Paper 1)

Lesson No. 8: Applications of Integration II: Volume

2005

8 (c) (ii) Use integration methods to derive a formula for the volume of a cone.

Solution

Ex. Use integration methods to derive a formula for the volume of a cone.

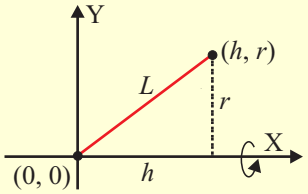
Solution

Slope of $L = \frac{r}{h} \Rightarrow$ Equation of L: $rx - hy = 0 \Rightarrow y = \frac{r}{h}x$

Volume of cone $V = \pi \int_0^r y^2\,dx$

$= \pi \frac{r^2}{h^2} \int_0^h x^2\,dx = \pi \frac{r^2}{3h^2}[x^3]_0^h = \pi \frac{r^2}{3h^2}[h^3 - 0]$

$= \frac{1}{3}\pi r^2 h$



2003

8 (c) (ii) Use integration methods to show that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.

Solution

Use integration methods to derive a formula for the volume of a sphere.

Solution

Rotate the circle C with centre (0, 0) and radius r about the X-axis.

Equation of C: $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2$

Volume of sphere $V = \pi \int_{-r}^r y^2\,dx = \pi \int_{-r}^r (r^2 - x^2)\,dx$

$= \pi[r^2 x - \frac{1}{3}x^3]_{-r}^r = \pi[(r^3 - \frac{1}{3}r^3) - (-r^3 + \frac{1}{3}r^3)] = \frac{4}{3}\pi r^3$