

INTEGRATION (Q 8, PAPER 1)

LESSON NO. 2: ALGEBRAIC INTEGRATION BY SUBSTITUTION

2006

8 (b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$

SOLUTION

$$I = \int_1^2 x(1+x^2) dx$$

1. Let $u = 1+x^2$.

2. $du = 2x dx \Rightarrow \frac{1}{2} du = x dx$

3. $x=1 \Rightarrow u=1+(1)^2=2$ and $x=2 \Rightarrow u=1+(2)^2=5$

$$4. I = \frac{1}{2} \int_2^5 u^3 du$$

$$5. \therefore I = \frac{1}{2} \left[\frac{1}{4} u^4 \right]_2^5 = \frac{1}{8} [u^4]_2^5 = \frac{1}{8} [625 - 16] = \frac{609}{8}$$

STEPS

1. Let u equal the more complicated function.
2. Differentiate u with respect to x .
3. Change the limits from x to u .
4. Make the substitution.
5. Evaluate the integral I .

2005

8 (b) Evaluate (i) $\int_1^4 \frac{2x+1}{x^2+x+1} dx$

SOLUTION

STEPS

1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x .
3. Change the limits from x to u .
4. Make the substitution.
5. Evaluate the integral I .

$$I = \int_1^4 \frac{2x+1}{x^2+x+1} dx$$

1. Let $u = x^2 + x + 1$.

2. $\Rightarrow du = (2x+1)dx$

3. $x=1 \Rightarrow u=1^2+1+1=3$ and $x=4 \Rightarrow u=4^2+4+1=21$.

$$4. \therefore I = \int_3^{21} \frac{du}{u}$$

$$5. \Rightarrow I = [\ln u]_3^{21} = \ln 21 - \ln 3 = \ln \frac{21}{3} = \ln 7$$

$$\int \frac{1}{x} dx = \ln x + c \quad \dots\dots \textcircled{2}$$

2003

8 (b) (i) Evaluate $\int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$

SOLUTION

STEPS

1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x .
3. Change the limits from x to u .
4. Make the substitution.
5. Evaluate the integral I .

$$I = \int_0^1 \frac{2x}{\sqrt{1+x^2}} dx$$

1. Let $u = 1 + x^2$
2. $\Rightarrow du = 2x dx$
3. $x = 0 \Rightarrow u = 1 + (0)^2 = 1$ and $x = 1 \Rightarrow u = 1 + (1)^2 = 2$

$$4. \therefore I = \int_1^2 \frac{du}{\sqrt{u}} = \int_1^2 u^{-\frac{1}{2}} du$$

$$5. \Rightarrow I = \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^2 = 2[\sqrt{u}]_1^2 = 2[\sqrt{2} - 1]$$

2002

8 (b) Evaluate (i) $\int_2^7 \frac{2x-4}{x^2-4x+29} dx$

SOLUTION

$$I = \int_2^7 \frac{2x-4}{x^2-4x+29} dx$$

STEPS

1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x .
3. Change the limits from x to u .
4. Make the substitution.
5. Evaluate the integral I .

$$1. \text{ Let } u = x^2 - 4x + 29$$

$$2. \Rightarrow du = (2x - 4)dx$$

$$3. x = 2 \Rightarrow u = 2^2 - 4(2) + 29 = 25 \text{ and } x = 7 \Rightarrow u = 7^2 - 4(7) + 29 = 50$$

$$4. \therefore I = \int_{25}^{50} \frac{du}{u}$$

$$5. \Rightarrow I = [\ln u]_{25}^{50} = \ln 50 - \ln 25 = \ln \frac{50}{25} = \ln 2$$

2001

8 (b) Evaluate (ii) $\int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}} dx$.

SOLUTION

$$I = \int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}} dx$$

STEPS

1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x .
3. Change the limits from x to u .
4. Make the substitution.
5. Evaluate the integral I .

1. Let $u = x^2 + 8x + 1$

2. $du = (2x + 8)dx = 2(x + 4)dx \Rightarrow \frac{1}{2} du = (x + 4)dx$

3. $x = 0 \Rightarrow u = (0)^2 + 8(0) + 1 = 1$ and $x = 4 \Rightarrow u = (4)^2 + 8(4) + 1 = 49$

4. $\therefore I = \frac{1}{2} \int_1^{49} \frac{du}{\sqrt{u}} = \frac{1}{2} \int_1^{49} u^{-\frac{1}{2}} du$

5. $\Rightarrow I = \frac{1}{2} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^{49} = [u^{\frac{1}{2}}]_1^{49} = [\sqrt{49} - \sqrt{1}] = [7 - 1] = 6$