DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

LESSON NO. 7: IMPLICIT DIFFERENTIATION

2005

7 (b) (ii) Find the slope of the tangent to the curve $xy^2 + y = 6$ at the point (1, 2).

SOLUTION

$$xy^2 + y = 6 \Rightarrow \left\{ x(2y)\frac{dy}{dx} + y^2(1) \right\} + 1\frac{dy}{dx} = 0 \text{ [Notice the product]}$$

$$\Rightarrow 2xy\frac{dy}{dx} + y^2 + \frac{dy}{dx} = 0 \Rightarrow (2xy+1)\frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^2}{(2xy+1)}$$

$$\left(\frac{dy}{dx}\right)_{(1,\,2)} = -\frac{2^2}{(2(1)(2)+1)} = -\frac{4}{4+1} = -\frac{4}{5}$$

2004

7 (c) Given that $x = \dfrac{e^{2y}-1}{e^{2y}+1}$,

(i) show that $e^{2y} = \dfrac{1+x}{1-x}$

(ii) show that $\dfrac{dy}{dx}$ can be expressed in the form $\dfrac{p}{1-x^p}$, $p, q \in \mathbf{N}$.

SOLUTION

7 (c) (i)

$$x = \frac{e^{2y}-1}{e^{2y}+1} \Rightarrow x(e^{2y}+1) = e^{2y} - 1 \Rightarrow xe^{2y} + x = e^{2y} - 1$$

$$\Rightarrow xe^{2y} - e^{2y} = -1 - x \Rightarrow e^{2y}(x-1) = -1 - x \Rightarrow e^{2y} = \frac{-1-x}{x-1} = \frac{1+x}{1-x}$$

7 (c) (ii)

$$e^{2y} = \frac{1+x}{1-x} \Rightarrow 2e^{2y}\frac{dy}{dx} = \frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$$

$$\Rightarrow 2\left(\frac{1+x}{1-x}\right)\frac{dy}{dx} = \frac{1-x+1-x}{(1-x)^2} \Rightarrow 2\left(\frac{1+x}{1-x}\right)\frac{dy}{dx} = \frac{2}{(1-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1-x)(1+x)} = \frac{1}{1-x^2}$$

2003

7 (b) (ii) Given that $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$, find the value of $\frac{dy}{dx}$ at the point $(2, -3)$.

Solution

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{6} \Rightarrow x^{-1}+y^{-1}=\frac{1}{6}$$

$$\Rightarrow -1x^{-2}-1y^{-2}\frac{dy}{dx}=0 \Rightarrow -x^{-2}=y^{-2}\frac{dy}{dx} \Rightarrow -\frac{x^{-2}}{y^{-2}}=\frac{dy}{dx} \Rightarrow -\frac{y^2}{x^2}=\frac{dy}{dx}$$

$$\left(\frac{dy}{dx}\right)_{(2,-3)}=-\frac{(-3)^2}{2^2}=-\frac{9}{4}$$

2002

7 (a) Find the slope of the tangent to the curve $9x^2+4y^2=40$ at the point (2, 1).

Solution

7 (a)

$$9x^2+4y^2=40 \Rightarrow 18x+8y\frac{dy}{dx}=0 \Rightarrow 8y\frac{dy}{dx}=-18x$$

$$\Rightarrow \frac{dy}{dx}=-\frac{18x}{8y}=-\frac{9x}{4y}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,1)}=-\frac{9(2)}{4(1)}=-\frac{18}{4}=-\frac{9}{2}$$