# DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

## LESSON NO. 4: LOGARITHMIC DIFFERENTIATION

**2006**

7 (c) Given $y = \ln\left(\dfrac{3+x}{\sqrt{9-x^2}}\right)$, find $\dfrac{dy}{dx}$ and express it in the form $\dfrac{a}{b-x^n}$.

### SOLUTION

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$ ....... 8

REMEMBER IT AS:

One over the function inside the log × Differentiation of function

**STEPS**
1. Using log properties break up the log function.
2. Differentiate each log.
3. Tidy up the algebra at the end.

**1.** $y = \ln\left(\dfrac{3+x}{\sqrt{9-x^2}}\right) = \ln(3+x) - \frac{1}{2}\ln(9-x^2)$ [Using log properties on page 36]

**2.** $\dfrac{dy}{dx} = \dfrac{1}{3+x} + \dfrac{2x}{2(9-x^2)} = \dfrac{1}{3+x} + \dfrac{x}{9-x^2}$

**3.** $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{(3+x)} + \dfrac{x}{(3+x)(3-x)} = \dfrac{1(3-x)+x}{(3+x)(3-x)} = \dfrac{3}{9-x^2}$

**2003**

7 (c) (i)Given that $y = \ln\dfrac{1+x^2}{1-x^2}$ for $0 < x < 1$, find $\dfrac{dy}{dx}$ and write your answer in the form

$\dfrac{kx}{1-x^k}$ where $k \in \mathbf{N}$.

### SOLUTION

**1.** $y = \ln\left(\dfrac{1+x^2}{1-x^2}\right) \Rightarrow y = \ln(1+x^2) - \ln(1-x^2)$

**2.** $\dfrac{dy}{dx} = \dfrac{2x}{(1+x^2)} + \dfrac{2x}{(1-x^2)}$

**3.** $\Rightarrow \dfrac{dy}{dx} = \dfrac{2x(1-x^2)+2x(1+x^2)}{(1+x^2)(1-x^2)} = \dfrac{2x-2x^3+2x+2x^3}{(1+x^2)(1-x^2)} = \dfrac{4x}{(1-x^4)}$