

**DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)**

**LESSON NO. 2: TRIGONOMETRIC DIFFERENTIATION**

**2005**

6 (b) Let  $y = \frac{1 - \cos x}{1 + \cos x}$ .

Show that  $\frac{dy}{dx} = t + t^3$ , where  $t = \tan\left(\frac{x}{2}\right)$ .

**SOLUTION**

Use the quotient rule:  $y = \frac{1 - \cos x}{1 + \cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x + \cos x \sin x + \sin x - \cos x \sin x}{(1 + \cos x)^2} = \frac{2 \sin x}{(1 + \cos x)^2}$$

The second part of this question involves proving a trig identity.

You are required to prove that  $\frac{dy}{dx} = t + t^3 \Rightarrow \frac{2 \sin x}{(1 + \cos x)^2} = \tan\left(\frac{x}{2}\right) + \tan^3\left(\frac{x}{2}\right)$

Deal with half angles: Let  $A = \frac{x}{2} \Rightarrow x = 2A$

*LHS*

$$\begin{aligned} \frac{2 \sin 2A}{(1 + \cos 2A)^2} &= \frac{4 \sin A \cos A}{(1 + \cos^2 A - \sin^2 A)^2} \\ &= \frac{4 \sin A \cos A}{(2 \cos^2 A)^2} = \frac{4 \sin A \cos A}{4 \cos^4 A} \\ &= \frac{\sin A}{\cos^3 A} \end{aligned}$$

*RHS*

$$\begin{aligned} \tan A + \tan^3 A &= \tan A(1 + \tan^2 A) \\ &= \tan A(\sec^2 A) = \frac{\sin A}{\cos A} \times \frac{1}{\cos^2 A} \\ &= \frac{\sin A}{\cos^3 A} \end{aligned}$$

**THE QUOTIENT RULE:** If  $y = \frac{u}{v}$  then:

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots\dots \mathbf{4}$$

**2003**

7 (a) Differentiate each of the following with respect to  $x$ :

(i)  $\cos^4 x$

7 (c) (ii) Given that  $f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi)$ , find the derivative of  $f(\theta)$  and express it in the form  $\cos n\theta$  where  $n \in \mathbf{Z}$ .

**SOLUTION**

**7 (a) (i)**

$$y = \cos^4 x = (\cos x)^4 \Rightarrow \frac{dy}{dx} = 4(\cos x)^3(-\sin x) = -4\cos^3 x \sin x$$

**7 (c) (ii)**

$$f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi) = \sin(\theta + 180^\circ)\cos(\theta - 180^\circ)$$

$\sin(\theta + 180^\circ) = -\sin \theta$  and  $\cos(\theta - 180^\circ) = -\cos \theta$  [You can work these out by expanding compound angles using the formulae on page 9 or by using ASTC and recognising them as well-behaved angles.]

$$\Rightarrow f(\theta) = (-\sin \theta)(-\cos \theta) = \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$$

$$\therefore f'(\theta) = \frac{1}{2}(2 \cos 2\theta) = \cos 2\theta$$

**2002**

6 (b) (ii) Given  $y = 2x - \sin 2x$ , find  $\frac{dy}{dx}$ . Give your answer in the form  $k \sin^2 x$ , where  $k \in \mathbf{Z}$ .

**SOLUTION**

$$y = 2x - \sin 2x \Rightarrow \frac{dy}{dx} = 2 - 2\cos 2x = 2(1 - \cos 2x)$$

$$\Rightarrow \frac{dy}{dx} = 2(1 - \cos^2 x + \sin^2 x) = 2(2 \sin^2 x) = 4 \sin^2 x$$

**2001**

7 (b) (ii) Given that  $y = \sin x \cos x$ , find  $\frac{dy}{dx}$  and express it in the form  $\cos nx$  where  $n \in \mathbf{Z}$ .

**SOLUTION**

$$y = \sin x \cos x = \frac{1}{2} \sin 2x$$

$\sin 2A = 2 \sin A \cos A$	.....	<b>13</b>
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$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(2 \cos 2x) = \cos 2x$$