DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

LESSON NO. 6: PARAMETRIC DIFFERENTIATION

2006

7 (b) The parametric equations of a curve are:

$$x = 3\cos\theta - \cos^3\theta$$
$$y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \tfrac{\pi}{2}.$$

(i) Find $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$.

(ii) Hence show that $\dfrac{dy}{dx} = \dfrac{-1}{\tan^3\theta}$.

2005

7 (b) (i) The parametric equations of a curve are:

$$x = 8 + \ln t^2$$
$$y = \ln(2 + t^2), \text{ where } t > 0.$$

Find $\dfrac{dy}{dx}$ in terms of t and calculate its value at $t = \sqrt{2}$.

2004

7 (b) The parametric equations of a curve are:

$$x = 2\theta - \sin 2\theta$$
$$y = 1 - \cos 2\theta, \text{ where } 0 < \theta < \pi.$$

(i) Find $\dfrac{dy}{dx}$.

(ii) Show that the tangent to the curve at $\theta = \frac{\pi}{6}$ is perpendicular to the tangent at $\theta = \frac{2\pi}{3}$.

2003

7 (b) (i) The parametric equations of a curve are:

$$x = \cos t + t\sin t$$
$$y = \sin t - t\cos t \text{ where } 0 < t < \tfrac{\pi}{2}.$$

Find $\dfrac{dy}{dx}$ and write your answer in its simplest form.

2002

7 (b) (ii) The parametric equations of a curve are $x = \ln(1+t^2)$ and $y = \ln 2t$, where

$t \in \mathbf{R}$, $t > 0$. Find the value of $\frac{dy}{dx}$ when $t = \sqrt{5}$.

2001

6 (c) Let $x = t^2 e^t$ and $y = t + 2\ln t$ for $t > 0$.

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in terms of t.

(ii) Hence, show that $\frac{dy}{dx} = \frac{1}{x}$.

Answers

2006 7 (b) (i) $\frac{dy}{d\theta} = 3\cos^3\theta$, $\frac{dx}{d\theta} = -3\sin^3\theta$

2005 7 (b) (i) $\frac{dy}{dx} = \frac{t^2}{2+t^2}$, $\left(\frac{dy}{dx}\right)_{t=\sqrt{2}} = \frac{1}{2}$

2004 7 (b) (i) $\frac{dy}{dx} = \frac{\sin 2\theta}{1-\cos 2\theta}$

2003 7 (b) (i) $\frac{dy}{dx} = \tan t$

2002 (b) (ii) $\left(\frac{dy}{dx}\right)_{t=\sqrt{5}} = \frac{3}{5}$

2001 6 (c) (i) $\frac{dx}{dt} = te^t(t+2)$, $\frac{dy}{dt} = 1 + \frac{2}{t}$