DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2005

6 (a) Differentiate with respect to x

(i) $(1+7x)^3$ (ii) $\sin^{-1}\left(\frac{x}{5}\right)$.

(b) Let $y=\frac{1-\cos x}{1+\cos x}$.

Show that $\frac{dy}{dx}=t+t^3$, where $t=\tan\left(\frac{x}{2}\right)$.

(c) The equation of a curve is $y=\frac{x}{x-1}$, where $x\neq 1$.

(i) Show that the curve has no local maximum or local minimum point.

(ii) Write down the equations of the asymptotes and hence sketch the curve.

(iii) Show that the curve is its own image under the symmetry in the point of intersection of the asymptotes.

7 (a) Find from first principles the derivative of x^2 with respect to x.

(b) (i) The parametric equations of a curve are:

$$x=8+\ln t^2$$
$$y=\ln(2+t^2),\ \text{where } t>0.$$

Find $\frac{dy}{dx}$ in terms of t and calculate its value at $t=\sqrt{2}$.

(ii) Find the slope of the tangent to the curve $xy^2+y=6$ at the point (1, 2).

(c) (i) Write down a quadratic equation whose roots are $\pm\sqrt{k}$.

(ii) Hence use the Newton-Raphson method to show that the rule $u_{n+1}=\frac{(u_n)^2+k}{2u_n}$ can be used to find increasingly accurate approximations for $\sqrt{k}$.

(iii) Using the above rule and taking $\frac{3}{2}$ as the first approximation for $\sqrt{3}$, find the third approximation, as a fraction.

ANSWERS

6 (a) (i) $21(1+7x)^2$ (ii) $\dfrac{1}{\sqrt{25-x^2}}$

6 (c) (ii) $x = 1$, $y = 1$

7 (b) (i) $\dfrac{dy}{dx} = \dfrac{t^2}{2+t^2}$, $\left(\dfrac{dy}{dx}\right)_{t=\sqrt{2}} = \dfrac{1}{2}$ (ii) $-\dfrac{4}{5}$

7 (c) (i) $x^2 - k = 0$ (iii) $\frac{97}{56}$