Complex Numbers & Matrices (Q 3, Paper 1)

Solutions No. 2: Complex Number Equations

2006

3 (a) Given that $z=2+i,$ where $i^2=-1,$ find the real number d such that $z+\dfrac{d}{z}$ is real.

Solution

3 (a)

$$z+\frac{d}{z}=2+i+\frac{d}{2+i}$$

$$\Rightarrow 2+i+\frac{d}{(2+i)}\times\frac{(2-i)}{(2-i)}=2+i+\frac{2d-id}{5}=2+\frac{2d}{5}+i-\frac{id}{5}$$

As this number is real $\Rightarrow \left(1-\dfrac{d}{5}\right)i=0\Rightarrow 1=\dfrac{d}{5}\Rightarrow d=5$

2005

3 (b) Solve the quadratic equation $2iz^2+(6+2i)z+(3-6i)=0,$ where $i^2=-1.$

Solution

Solve $2iz^2+(6+2i)z+(3-6i)=0$

$a=2i$

$b=(6+2i)$

$c=(3-6i)$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(6+2i)\pm\sqrt{(6+2i)^2-4(2i)(3-6i)}}{4i}$$

$$=\frac{-(6+2i)\pm\sqrt{36+24i+4i^2-24i+48i^2}}{4i}=\frac{-(6+2i)\pm\sqrt{36-52}}{4i}$$

$$=\frac{-(6+2i)\pm\sqrt{-16}}{4i}=\frac{-6-2i\pm4i}{4i}=\frac{-6+2i}{4i},\ \frac{-6-6i}{4i}$$

$$=\frac{-3+i}{2i},\ \frac{-3-3i}{2i}=\frac{1+3i}{2},\ \frac{-3+3i}{2}$$

2004

3 (a) Find the real numbers p and q such that $2(p+iq)+i(p-iq)=5+i,$ where $i^2=-1.$

Solution

3 (a)

$2(p+iq)+i(p-iq)=5+i\Rightarrow(2p-i^2q)+(p+2q)=5+i$

$\Rightarrow(2p+q)+(p+2q)=5+i$ [Equate the real parts and the imaginary parts.]

$\Rightarrow 2p+q=5$ and $p+2q=1$

Solving simultaneously: $p=3,\ q=-1$

2003

3 (b) (ii) k is a real number such that $\dfrac{-1+i\sqrt{3}}{-4\sqrt{3}-4i}=ki$. Find k.

Solution

$$\frac{-1+i\sqrt{3}}{-4\sqrt{3}-4i}=ki \Rightarrow -1+i\sqrt{3}=ki(-4\sqrt{3}-4i)$$

$$\Rightarrow -1+i\sqrt{3}=4k-4k\sqrt{3}i$$

Equating the real parts $\Rightarrow -1=4k \Rightarrow k=-\frac{1}{4}$

2002

3 (b) (ii) w is a complex number such that $w\bar{w}-2iw=7-4i$, where $\bar{w}$ is the complex conjugate of w.
Find two possible values of w. Express each in the form $p+qi$, where $p, q \in \mathbf{R}$.

Solution

Let $w=p+qi$ and $\bar{w}=p-qi$.

$$w\bar{w}-2iw=7-4i \Rightarrow (p+qi)(p-qi)-2i(p+qi)=7-4i$$

$$\Rightarrow p^2+q^2-2ip-2qi^2=7-4i \Rightarrow (p^2+q^2+2q)-2ip=7-4i$$

Equate the real and imaginary parts $\Rightarrow (p^2+q^2+2q)=7$ and $-2p=-4 \Rightarrow p=2$

$$\Rightarrow (2)^2+q^2+2q=7 \Rightarrow q^2+2q-3=0 \Rightarrow (q+3)(q-1)=0$$

$$\Rightarrow q=-3,\ 1$$

Answers: $2-3i,\ 2+i$

2001

3 (c) (ii) Show that z^2-16 is a factor of $z^3+(1+i)z^2-16z-16(1+i)$ and hence, find the three roots of $z^3+(1+i)z^2-16z-16(1+i)=0$.

Solution

To show that z^2-16 is a factor of $z^3+(1+i)z^2-16z-16(1+i)$, divide it in.

$$\begin{array}{r|l}
 & z+(1+i) \\
\hline
z^2-16 & z^3+(1+i)z^2-16z-16(1+i) \\
 & \mp z^3 \qquad\qquad \pm 16z \\
\hline
 & \qquad (1+i)z^2 \qquad -16(1+i) \\
 & \qquad \mp(1+i)z^2 \qquad \pm 16(1+i) \\
\hline
 & \qquad\qquad 0
\end{array}$$

As the remainder is zero, z^2-16 is a factor.

$$z^3+(1+i)z^2-16z-16(1+i)=(z^2-16)(z+(1+i))=(z+4)(z-4)(z+(1+i))=0$$

$$\Rightarrow z=-4,\ 4,\ -(1+i)$$