Complex Numbers & Matrices (Q 3, Paper 1)

Solutions No. 1: Complex Number Algebra

2004

3 (b) (ii) $w_1 = a + ib$ and $w_2 = c + id$. Prove that $\overline{(w_1 w_2)} = (\overline{w_1})(\overline{w_2})$, where $\overline{w}$ is the complex conjugate w.

Solution

LHS

$\overline{(w_1 w_2)} = \overline{(a+ib)(c+id)}$

$= \overline{(ac-bd)+(ad+bc)i}$

$= (ac-bd)-(ad+bc)i$

RHS

$(\overline{w_1})(\overline{w_2}) = (\overline{a+ib})(\overline{c+id})$

$= (a-ib)(c-id)$

$= (ac-bd)-(ad+bc)i$

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3 (b) (i) Given that $z = 2 - i$, calculate $|z^2 - z + 3|$ where $i^2 = -1$.

Solution

$|z^2 - z + 3| = |(2-i)^2 - (2-i) + 3| = |4 - 4i + i^2 - 2 + i + 3|$

$= |4 - 3i| = \sqrt{(4)^2 + (-3)^2} = 5$

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3 (a) Let $u = \dfrac{1+3i}{3+i}$ where $i^2 = -1$.

(i) Express u is the form $a + ib$ where $a, b \in \mathbf{R}$.

(ii) Evaluate $|u|$.

Solution

3 (a) (i)

$$u = \frac{1+3i}{3+i} = \frac{1+3i}{3+i} \times \frac{3-i}{3-i} = \frac{3 - i + 9i - 3i^2}{10} = \frac{6+8i}{10} = \frac{3+4i}{5}$$

$= \frac{3}{5} + \frac{4}{5} i$

3 (a) (ii)

$$u = \tfrac{3}{5} + \tfrac{4}{5} i \Rightarrow |u| = \sqrt{(\tfrac{3}{5})^2 + (\tfrac{4}{5})^2} = \sqrt{\tfrac{9}{25} + \tfrac{16}{25}} = \sqrt{\tfrac{25}{25}} = 1$$