

ALGEBRA (Q 1 & 2, PAPER 1)

SOLUTIONS NO. 9: SIMULTANEOUS EQUATIONS

2005

1 (a) Solve the simultaneous equations:

$$\frac{x}{5} - \frac{y}{4} = 0$$
$$3x + \frac{y}{2} = 17$$

SOLUTION

$$\frac{x}{5} - \frac{y}{4} = 0 \text{ (}\times 20\text{)} \Rightarrow 4x - 5y = 0 \dots \text{(1)}$$

$$3x + \frac{y}{2} = 17 \text{ (}\times 2\text{)} \Rightarrow 6x + y = 34 \dots \text{(2)}$$

$$4x - 5y = 0 \dots \text{(1)}$$

$$6x + y = 34 \dots \text{(2)} \text{ (}\times -5\text{)}$$

$$4x - 5y = 0 \dots \text{(1)}$$

$$30x + 5y = 170 \dots \text{(2)}$$

$$\underline{34x} \quad = 170 \Rightarrow x = 5$$

Substituting this value of x into equation 1 $\Rightarrow 4(5) - 5y = 0 \Rightarrow 20 = 5y \Rightarrow y = 4$

ANSWER: $x = 5, y = 4$

2004

2 (a) Solve, without using a calculator, the following simultaneous equations:

$$3x + y + z = 0$$

$$x - y + z = 2$$

$$2x - 3y - z = 9$$

SOLUTION

Eliminate z :

$$3x + y + z = 0 \dots \text{(1)}$$

$$x - y + z = 2 \dots \text{(2)}$$

$$2x - 3y - z = 9 \dots \text{(3)}$$

$$5x - 2y = 9 \dots \text{(4)} \text{ (}\times -2\text{)}$$

$$3x - 4y = 11 \dots \text{(5)}$$

$$\text{Equation 1} + \text{3} \Rightarrow 5x - 2y = 9 \dots \text{(4)}$$

$$\text{Equation 2} + \text{3} \Rightarrow 3x - 4y = 11 \dots \text{(5)}$$

$$-10x + 4y = -18$$

$$\underline{3x - 4y} = 11$$

$$-7x = -7 \Rightarrow x = 1$$

Substituting this value of x into equation 4 $\Rightarrow 5(1) - 2y = 9 \Rightarrow -2y = 4 \Rightarrow y = -2$

Substituting these values of x and y into equation 1 $\Rightarrow 3(1) + (-2) + z = 0 \Rightarrow z = -1$

ANSWER: $x = 1, y = -2, z = -1$

2002

2 (a) Solve, without using a calculator, the following simultaneous equations:

$$x + 2y + 4z = 7$$

$$x + 3y + 2z = 1$$

$$-y + 3z = 8$$

SOLUTION

Eliminate x from equations **1** and **2**:

$$x + 2y + 4z = 7 \dots (1)$$

$$x + 3y + 2z = 1 \dots (2)$$

$$-y + 3z = 8 \dots (3)$$

$$x + 2y + 4z = 7 \dots (1)$$

$$x + 3y + 2z = 1 \dots (2) (\times -1)$$

$$x + 2y + 4z = 7$$

$$-x - 3y - 2z = -1$$

$$\hline -y + 2z = 6 \dots (4)$$

Now combine equations **3** and **4** to eliminate y :

$$-y + 3z = 8 \dots (3)$$

$$-y + 2z = 6 \dots (4) (\times -1)$$

$$-y + 3z = 8$$

$$y - 2z = -6$$

$$\hline z = 2$$

Substituting this value of z into equation **3**

$$\Rightarrow -y + 3(2) = 8 \Rightarrow y = -2$$

Substituting these values of y and z into equation **1**

$$\Rightarrow x + 2(-2) + 4(2) = 7 \Rightarrow x = 3$$

ANSWER: $x = 3, y = -2, z = 2$

2006

2 (a) Solve the simultaneous equations:

$$y = 2x - 5$$

$$x^2 + xy = 2$$

SOLUTION

Substitute the value of y in the linear equation into the quadratic equation.

$$x^2 + xy = 2 \Rightarrow x^2 + x(2x - 5) = 2 \Rightarrow x^2 + 2x^2 - 5x - 2 = 0$$

$$\Rightarrow 3x^2 - 5x - 2 = 0 \Rightarrow (3x + 1)(x - 2) = 0$$

$$x = -\frac{1}{3}, 2$$

$$x = -\frac{1}{3} \Rightarrow y = 2(-\frac{1}{3}) - 5 = -\frac{17}{3}$$

$$x = 2 \Rightarrow y = 2(2) - 5 = -1$$

ANSWER: $x = -\frac{1}{3}, 2; y = -\frac{17}{3}, -1$

2003

2 (a) Solve the simultaneous equations:

$$\begin{aligned}3x - y &= 8 \\ x^2 + y^2 &= 10\end{aligned}$$

SOLUTION

Substitute the value of y in the linear equation into the quadratic equation.

$$3x - y = 8 \Rightarrow y = 3x - 8$$

$$x^2 + y^2 = 10 \Rightarrow x^2 + (3x - 8)^2 = 10$$

$$\Rightarrow x^2 + 9x^2 - 48x + 64 - 10 = 0 \Rightarrow 10x^2 - 48x + 54 = 0$$

$$\Rightarrow 5x^2 - 24x + 27 = 0 \Rightarrow (5x - 9)(x - 3) = 0$$

$$\Rightarrow x = \frac{9}{5}, 3$$

$$x = \frac{9}{5} \Rightarrow y = 3\left(\frac{9}{5}\right) - 8 = -\frac{13}{5}$$

$$x = 3 \Rightarrow y = 3(3) - 8 = 1$$

ANSWER: $x = 3, \frac{9}{5}; y = 1, -\frac{13}{5}$

2001

2 (a) Solve the simultaneous equations:

$$\begin{aligned}x - y &= 0 \\ (x + 2)^2 + y^2 &= 10\end{aligned}$$

SOLUTION

From the linear equation $x = y$.

$$\text{Substituting for } y \text{ in the quadratic } \Rightarrow (x + 2)^2 + x^2 = 10 \Rightarrow x^2 + 4x + 4 + x^2 - 10 = 0$$

$$\Rightarrow 2x^2 + 4x - 6 = 0 \Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0 \Rightarrow x = -3, 1 \Rightarrow y = -3, 1$$

ANSWER: $x = -3, 1; y = -3, 1$