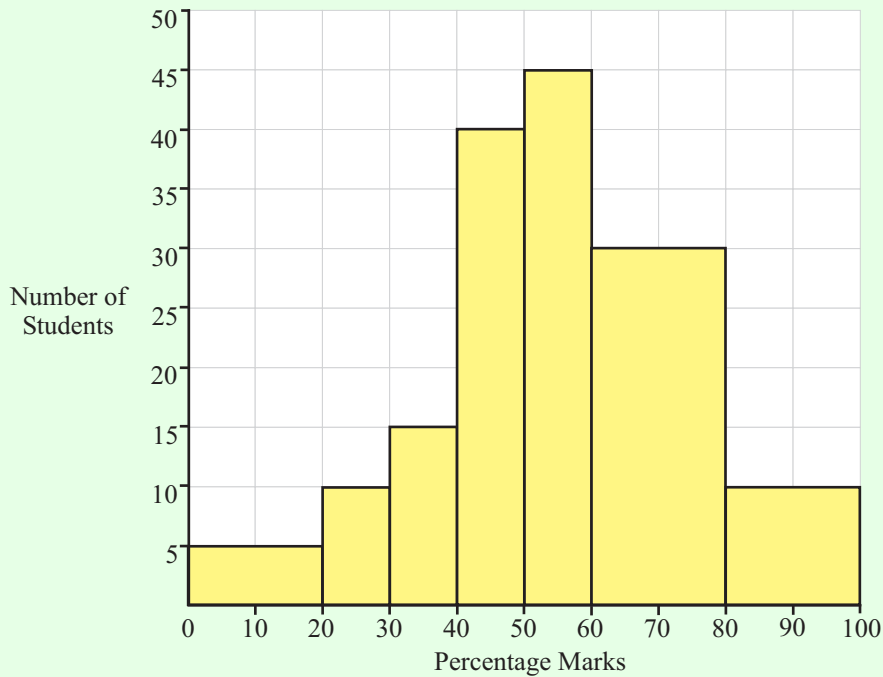**STATISTICS (Q 7, PAPER 2)**

1998

7 (a) Find the mean and the median of the following array of numbers:
2, 5, 7, 11, 15, 3, 6

(b) The distribution of percentage marks awarded to a group of 200 Leaving Certificate students in a particular subject is shown in the histogram below.

(i) If 45 students obtained between 50% and 60%, copy and complete the frequency table below.

Marks (%)	0 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 80	80 – 100
Frequency					45		

(ii) What is the greatest possible number of students who could have obtained a grade C or better (i.e. mark ≥ 55)?

(c) The following table shows the sizes, in hectares, of 20 farms in a particular area:

No. of hectares	15 – 45	45 – 75	75 – 105	105 – 195
Number of farms	1	4	8	7

By taking the data at mid-interval values, calculate
(i) the mean number of hectares per farm

(ii) the standard deviation, correct to the nearest hectare.

SOLUTION

7 (a)

> The **MEAN** or **AVERAGE** of a set of numbers is calculated by adding the numbers together and dividing by the number of numbers.
>
> $$\text{Mean} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$
>
> The mean is denoted by $\bar{x}$.

$$\bar{x} = \frac{2+5+7+11+15+3+6}{7}$$

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N} \quad \ldots\ldots \; \textbf{1}$$

$$\Rightarrow \bar{x} = \frac{49}{7} = 7$$

> The **MEDIAN** is the middle number when you line up all the numbers in increasing order.

2, 3, 5, **6**, 7, 11, 15

You can see that 6 is the median number.

7 (b) (i)

Area (Frequency) = Base × Height

For the given interval: $45 = \text{Base} \times 45 \Rightarrow \text{Base} = 1$

As can be seen from the histogram, the given interval occupies 1 marked unit along the horizontal axis. Therefore, an interval of 20 has a base of 2 and an interval of 10 has a base of 1.

INTERVAL No. 1: Area = 2 × 5 = 10
INTERVAL No. 2: Area = 1 × 10 = 10
INTERVAL No. 3: Area = 1 × 15 = 15
INTERVAL No. 4: Area = 1 × 40 = 40
INTERVAL No. 5: Given in the table
INTERVAL No. 6: Area = 2 × 30 = 60
INTERVAL No. 7: Area = 2 × 10 = 20

Now complete the table:

Marks (%)	0 − 20	20 − 30	30 − 40	40 − 50	50 − 60	60 − 80	80 − 100
Frequency	10	10	15	40	45	60	20

7 (b) (ii)

There are 45 students who obtained a mark between 50 and 60. It is possible that all 45 students in this group could have obtained 55 marks or more. Therefore, the greatest possible number of students who have obtained a grade C or better
= 45 + 60 + 20 = 125 students.

7 (c)

If you are asked to find the mean and standard deviation of a frequency distribution, set it out in a table as shown.

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_N x_N}{f_1 + f_2 + \dots + f_N} = \frac{\sum fx}{\sum f}$$ **2**

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}}$$ **5**

x	f	fx	d	d^2	fd^2
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
	$\sum f$	$\sum fx$			$\sum fd^2$

Work out the mean first. Then work out d using $d = (x - \bar{x})$. Finally, calculate the standard deviation.

x	f	fx	d	d^2	fd^2
30	1	30	-72	5184	5184
60	4	240	-42	1764	7056
90	8	720	-12	144	1152
150	7	1050	48	2304	16128
	20	2040			29520

Draw up a table in the way as shown on the left. The mid-interval values, x, are obtained by adding the class interval values together and dividing by two.

(i) $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{2040}{20} = 102$

(ii) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f}} = \sqrt{\dfrac{29520}{20}} = 38$