Statistics (Q 7, Paper 2)

Lesson No. 6: Cumulative Frequency

2007

7 (b) The table below shows the time, in minutes, that customers were waiting to be served in a restaurant.

Time (minutes)	< 5	< 10	< 15	< 20	< 25
Number of customers	5	20	70	110	120

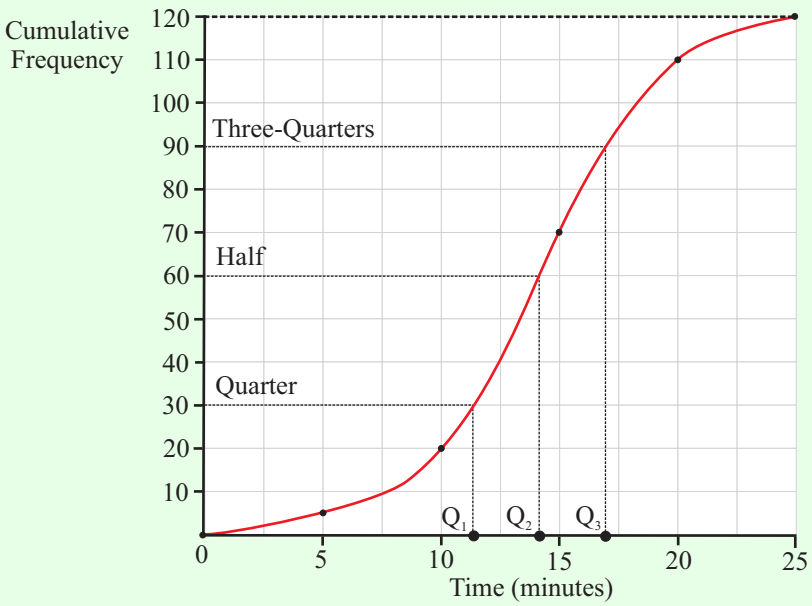
(i) Draw a cumulative frequency curve (ogive).

(ii) Use your curve to estimate the median waiting time.

(iii) Use your curve to estimate the interquartile range.

Solution

7 (b) (i)



7 (b) (ii)

Median (Q_2): The total number of customerss was 120. Half of this number is 60. The median is 14 minutes.

7 (b) (iii)

The lower quartile (Q_1): Go to 30 on the vertical axis (one-quarter of the customers). The lower quartile is 11 minutes.

The upper quartile (Q_3): Go to 90 on the vertical axis (three-quarters of the students). The upper quartile is 17 minutes.

The interquartile range: $Q_3 - Q_1 = 17 - 11 = 6$ minutes

2006

7 (b) The number of new cars in various price ranges sold by a retailer in one month is recorded in the following table:

Price (€1000's)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 50
Number sold	5	15	25	15	20

[Note: 15 – 20 means at least 15 but less than 20, etc.]

(i) Draw a histogram to represent the data.

(ii) By taking the data at the mid-interval values, calculate the mean price per car.

(iii) Copy and complete the following cumulative frequency table:

Price (€1000's)	< 15	< 20	< 25	< 30	< 50
Number sold					

(iv) Draw the cumulative frequency curve (ogive).

(v) Using your curve, estimate how many of the cars sold were priced between the mean and the median.

SOLUTION

7 (b) (i)

Price (€1000's)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 50
Number sold	5	15	25	15	20

Each column in the table is represented by a rectangular box. The area of the box corresponds to the frequency (no. of students).

Area (No. of students) = Base × Height = Frequency

Look at the prices. Pick out the smallest interval (10 – 15) and make this base one unit. Therefore the interval 30 – 50 has a base of 4 units. Divide the base into the area (frequency) to get the height of a box.

Draw a new table:

Interval (Prices)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 50
Frequency (No. sold)	5	15	25	15	20
Base	1	1	1	1	4
Height	5	15	25	15	5

Drawing the histogram:
Horizontal (*x*-axis) axis (Prices): Look at the intervals. The prices go from 0 to 50. The smallest interval (Base 1) is 5 so go up in 5's.
Vertical (*y*-axis) axis (No. sold): Always start at zero. The biggest number is the maximum height (i.e. 25).

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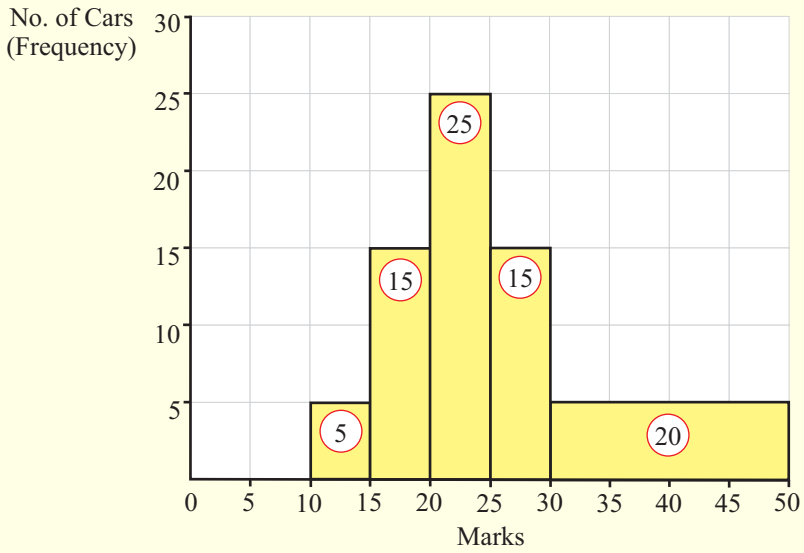


7 (b) (ii)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.
Ex. Class interval: 30 – 50

Mid-interval value: $\frac{30+50}{2} = 40$

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_Nx_N}{f_1 + f_2 + \ldots\ldots + f_N} = \frac{\sum fx}{\sum f} \quad \ldots\ldots \text{(2)}$$

x	f	fx
12.5	5	62.5
17.5	15	262.5
22.5	25	562.5
27.5	15	412.5
40.0	20	800
	80	2100

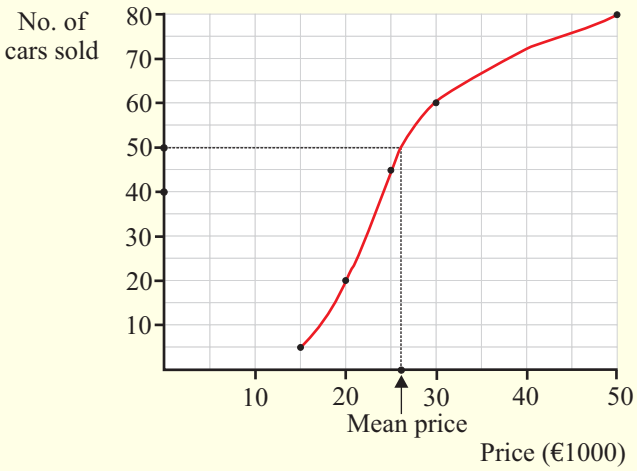
Mean price: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{2100}{80} = 26.25$

As the prices are in thousands of euro, the mean price is €26.25 × 1000 = €26,250

7 (b) (iii)

Price (€1000's)	< 15	< 20	< 25	< 30	< 50
Number sold	5	20	45	60	80

7 (b) (iv)



7 (b) (v)

Go to the mean price (26.25) on the horizontal axis. Draw a broken line up to the curve and across to the vertical axis. The number of cars sold at the mean price is 50.
The median number of cars sold is 40 (half of 80).
No. of cars sold priced between the mean and median = 50 – 40 = 10

2005

7 (c) A concert began at 8.00 p.m. The cumulative frequency table below gives the number of people in the concert hall at the times stated.

Time p.m.	7.10	7.20	7.30	7.40	7.50	8.00
Number of people	0	30	100	160	275	300

(i) Copy and complete the following frequency table to show the number of people who entered the hall during each time interval.

Time interval	7.10 – 7.20	7.20 – 7.30	7.30 – 7.40	7.40 – 7.50	7.50 – 8.00
Number of people					

(ii) In which interval does the median time of arrival lie?

(iii) In which time interval did the greatest number of people enter the concert hall?

(iv) What is the least number of people who could have been in the concert hall at 7.15 p.m?

SOLUTION

7 (c) (i)

To work out each value subtract the numbers in the cumulative frequency table.

Time interval	7.10 – 7.20	7.20 – 7.30	7.30 – 7.40	7.40 – 7.50	7.50 – 8.00
Number of people	30	70	60	115	25

7 (c) (ii)

There are 50 minutes from 7.10 to 8.00. The median is the half-way point which occurs after 25 minutes which is a time of 7.35. This occurs in the interval 7.30 – 7.40.

7 (c) (iii)

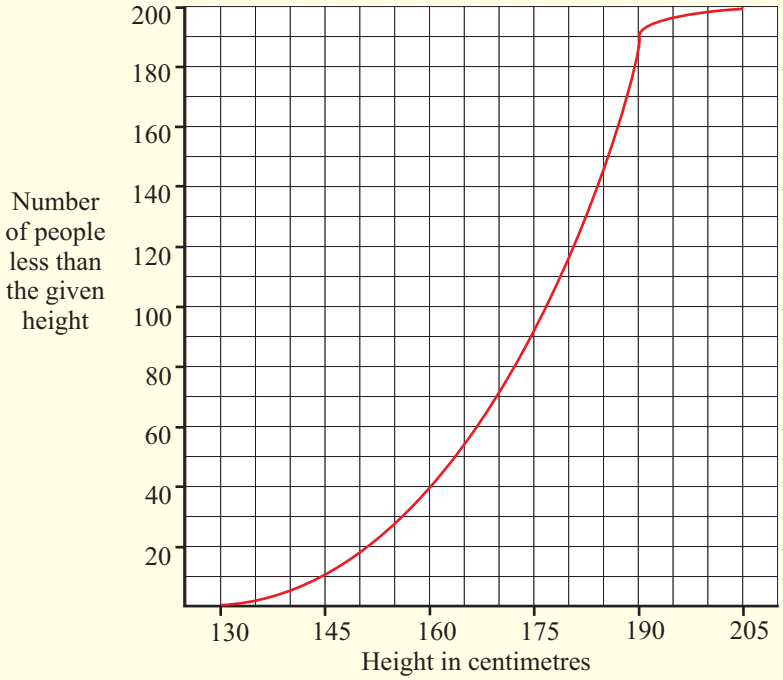
Between 7.40 – 7.50 the greatest number of 115 people arrived.

7 (c) (iv)

30 people arrived between 7.10 – 7.20. It is possible that all 30 people arrived after 7.15. Therefore, the least number of people who could have been in the concert hall at 7.15 is zero people.

2003

7 (a) The heights of 200 people are recorded to the nearest centimetre. The results are represented by the ogive below.



(i) Copy the cumulative frequency table below and use the ogive to complete it.

Height	<130	<145	<160	<175	<190	<205
Number of people	0					

(ii) Hence, copy and complete the following grouped frequency table:

Height	130 – 144	145 – 159	160 – 174	175 – 189	190 – 204
Number of people					

(iii) Using your grouped frequency table, and taking mid-interval values, find an estimate of the mean height.

(iv) Use the ogive to estimate the number of people who are taller than the mean.

SOLUTION

7 (a) (i)

Height	<130	<145	<160	<175	<190	<205
Number of people	0	10	40	90	190	200

7 (a) (ii)

Height	130 – 144	145 – 159	160 – 174	175 – 189	190 – 204
Number of people	10	30	50	100	10

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7 (a) (iii)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.
Ex. Class interval: 130 – 144

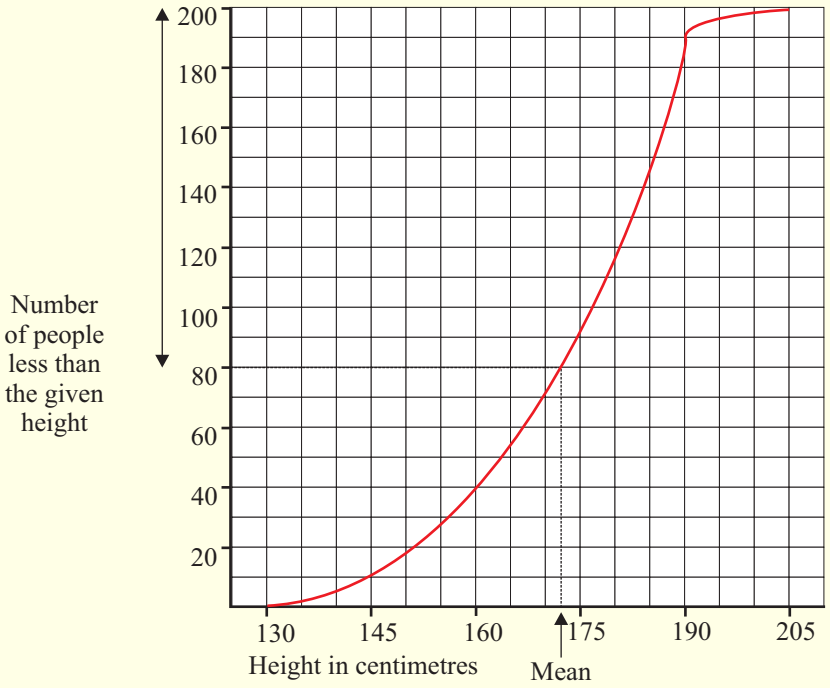
Mid-interval value: $\frac{130+144}{2}=137$

x	f	fx
137	10	1370
152	30	4560
167	50	8350
182	100	18200
197	10	1970
	200	34450

$$\bar{x}=\frac{f_1x_1+f_2x_2+.....+f_Nx_N}{f_1+f_2+.....+f_N}=\frac{\sum fx}{\sum f} \quad ..... \ (2)$$

Mean height: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{34450}{200}=172.25$

7 (a) (iv)



As you can see from the ogive, the number of people who are taller than the mean height = 200 – 80 = 120.

2002

7 (b) The following cumulative frequency table refers to the ages of 70 guests at a wedding:

Age (in years)	< 20	< 40	< 60	< 90
Number of guests	6	23	44	70

(i) Copy and complete the following frequency table:

Age (in years)	0 – 20	20 – 40	40 – 60	60 – 90
Number of guests				

[Note: 20 – 40 means 20 years old or more but less than 40 etc.]

(ii) Using mid-interval values, calculate the mean age of the guests.

(iii) What is the greatest number of guests who could have been over 65 years of age?

SOLUTION

7 (b) (i)

Age (in years)	0 – 20	20 – 40	40 – 60	60 – 90
Number of guests	6	17	21	26

7 (b) (ii)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.

Ex. Class interval: 20 – 40

Mid-interval value: $\frac{20+40}{2}=30$

x	f	fx
10	6	60
30	17	510
50	21	1050
75	26	1950
	70	3570

$$\bar{x}=\frac{f_1x_1+f_2x_2+.....+f_Nx_N}{f_1+f_2+.....+f_N}=\frac{\sum fx}{\sum f} \quad \text{.....} \ (2)$$

Mean age: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{3570}{70}=51$

7 (b) (iii)

There are 26 people aged between 60 and 90 years. Therefore, the greatest number of people aged over 65 years of age could be 26.

2001

7 (b) The following table shows the distribution of the amounts spent by 40 customers in a shop:

Amount Spent (IR£)	0 – 8	8 – 12	12 – 16	16 – 20	20 – 32
Number of Customers	2	9	13	10	6

[Note: IR£8 - IR£12 means IR£8 or over but less than IR£12 etc.]

(i) Taking mid-interval values, estimate the mean amount spent by the customers.

(ii) Copy and complete the following cumulative frequency table:

Amount Spent (IR£)	< 8	< 12	< 16	< 20	< 32
Number of Customers					

(iii) Draw a cumulative frequency curve (ogive).

(iv) Use your curve to estimate the number of customers who spent IR£25 or more.

SOLUTION

7 (b) (i)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.
Ex. Class interval: 8 – 12

Mid-interval value: $\frac{8+12}{2}=10$

x	f	fx
4	2	8
10	9	90
14	13	182
18	10	180
26	6	156
	40	616

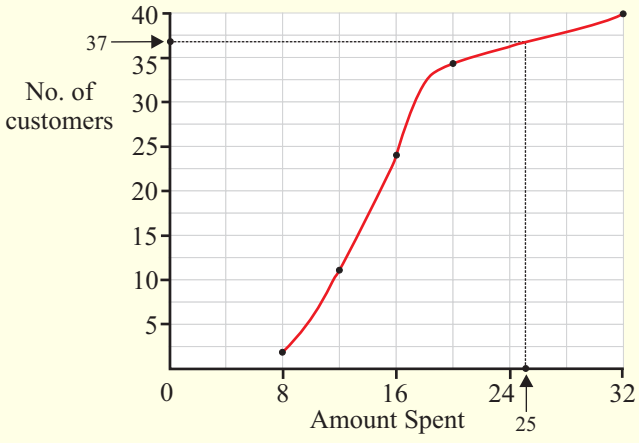
$$\bar{x}=\frac{f_1x_1+f_2x_2+.....+f_Nx_N}{f_1+f_2+.....+f_N}=\frac{\sum fx}{\sum f} \quad ..... \ (2)$$

Mean price: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{616}{40}=€15.40$

7 (b) (ii)

Amount Spent (IR£)	< 8	< 12	< 16	< 20	< 32
Number of Customers	2	11	24	34	40

7 (b) (iii)



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7 (b) (iv)

Go to £25 on the horizontal axis. Draw a line straight up until it meets the curve and then go across where, as you can see, there are 37 customers. Therefore, the number of customers who spent more that £35 is 40 – 37 = 3 customers.

2000

7 (c) The table below refers to the number of emergency calls recorded at a fire station each week for 52 weeks.

No. of emergency calls	0 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70
Number of weeks	6	8	11	12	7	5	3

(i) Copy and complete the following cumulative frequency table:

No. of emergency calls	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Number of weeks	6						52

(ii) Draw the cumulative frequency curve.

(iii) Use your graph to estimate the interquartile range.

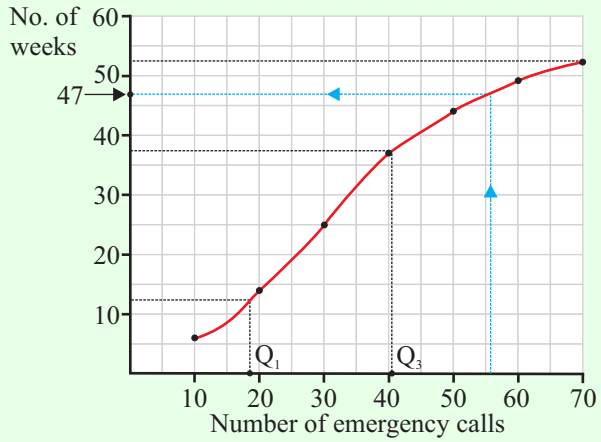
(iv) Use your graph to estimate the number of weeks during which more than 56 emergency calls were recorded.

SOLUTION

7 (c) (i)

No. of emergency calls	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Number of weeks	6	14	25	37	44	49	52

7 (c) (ii)



7 (c) (iii)

The lower quartile (Q_1): Go to 13 on the vertical axis (one-quarter of the number of weeks). The lower quartile is 19 calls.

The upper quartile (Q_3): Go to 39 on the vertical axis (three-quarters of the number of weeks). The upper quartile is 42 calls.

Interquartile range: $Q_3 - Q_1 = 42 - 19 = 23$

7 (c) (iv)
No. of weeks receiving more that 56 emergency calls: Go to 56 on the horizontal axis. You can see that you arrive at 47. Therefore the number of weeks is $= 52 - 47 = 5$ weeks.

1999

7 (b) The cumulative frequency table below shows the distribution of ages of 110 people living in an estate.

Age in years	≤ 5	≤ 10	≤ 20	≤ 35	≤ 50	≤ 60
Number of people	5	15	40	90	105	110

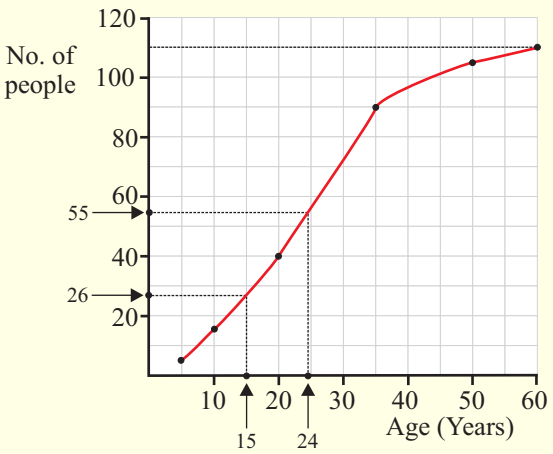
(i) Draw the cumulative frequency curve, putting number of people on the vertical axis.

(ii) Use your curve to estimate the median age.

(iii) Use your curve to estimate the number of people who are more than 15 years of age.

SOLUTION

7 (b) (i)



7 (b) (ii)
Half of the number of people is 55. Draw a horizontal line across to the curve and go straight down. You can read off the median age which is 24 years.

7 (b) (iii)
Go to 15 years of age on the horizontal axis. This age corresponds to 26 people. Therefore, the number of people who are more than 15 years of age = 110 – 26 = 84 people.

1997

7 (c) A new shop opened at 0900 hours. During the first hour of trading, customers were counted as they entered the shop. The following cumulative frequency table shows the number of customers who has entered before the given times:

Time	0910	0920	0930	0940	0950	1000
No. of customers	45	69	95	120	144	250

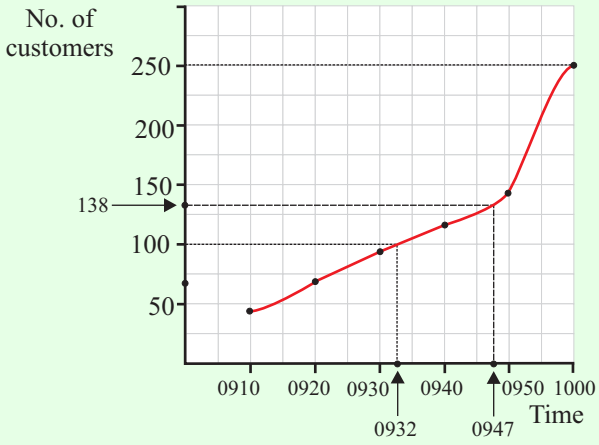
(i) Draw a cumulative frequency curve.

(ii) A photograph was taken of the 100th. customer as he or she entered the shop. Use your curve to estimate the time at which the photograph was taken.

(iii) Use your curve to estimate the number of customers who entered the shop during the 15 minutes immediately after the photograph was taken.

SOLUTION

7 (c) (i)



7 (c) (ii)

Go to 100 on the vertical axis. The corresponding value on the horizontal axis is a time of 0932.

7 (c) (iii)

15 minutes after 0932 is 0947. Go to 0947 on the horizontal axis. The corresponding value on the vertical axis is 138 customers. Therefore, the number of customers who entered the shop between 0932 and 0947 is 138 – 100 = 38 customers.

1996

7 (b) The cumulative frequency table below shows the number of minutes taken by 80 people to complete a crossword:

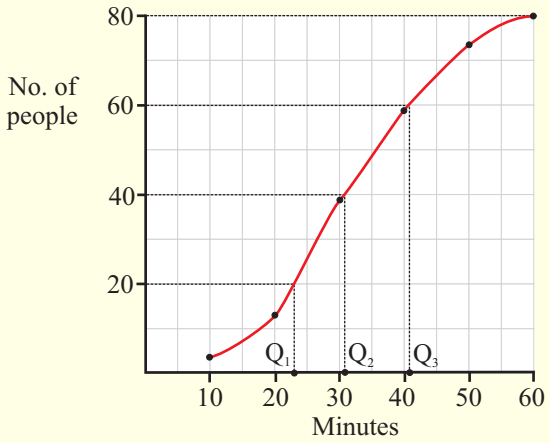
Minutes	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Cumulative Frequency	3	13	39	59	73	80

Draw a cumulative frequency curve.
Use your curve to estimate

(i) the median time to complete the crossword

(ii) the interquartile range.

SOLUTION



7 (b) (i)

Finding the Median (Q_2): The total number of pople who did the crossword was 80. Half of this number is 40. The median time as you can see from the graph is 31 minutes.

7 (b) (ii)

The lower quartile (Q_1): Go to 20 on the vertical axis (one-quarter of the people). The lower quartile is 23 minutes.

The upper quartile (Q_3): Go to 60 on the vertical axis (three-quarters of the people). The upper quartile is 41 minutes.

The interquartile range: $Q_3 - Q_1 = 41 - 23 = 18$ minutes.