STATISTICS (Q 7, PAPER 2)

LESSON NO. 5: HISTOGRAMS

2004

7   (b)   The following table shows the time in minutes spent by customers in a cafeteria.

Time in minutes	0 – 10	10 – 20	20 – 40	40 – 70
Number of customers	80	100	160	60

[Note that 10 – 20 means at least 10 but less than 20 minutes etc.]

(i)   Find the total number of customers.

(ii)   Draw a histogram to represent the data.

(iii)   By taking the data at the mid-interval values, calculate the mean number of minutes per customer.

(iv)   What is the greatest number of customers who could have spent more than 30 minutes in the cafeteria?

(v)   What is the least number of customers who could have spent more than 30 minutes in the cafeteria?

SOLUTION

7 (b) (i)

Total number of customers = 80 + 100 + 160 + 60 = 400

7 (b) (ii)

Time in minutes	0 – 10	10 – 20	20 – 40	40 – 70
Number of customers	80	100	160	60

Each column in the table is represented by a rectangular box. The area of the box corresponds to the frequency (no. of customers).

$$\text{Area (No. of students)} = \text{Base} \times \text{Height} = \text{Frequency}$$

Look at the times. Pick out the smallest interval (0 – 10) and make this base one unit. Therefore the interval 20 – 40 has a base of 2 units and the interval 40 – 70 has a base of 3 units. Divide the base into the area (frequency) to get the height of a box.

Draw a new table:

Interval (Time)	0 – 10	10 – 20	20 – 40	40 – 70
Frequency (No. of customers)	80	100	160	60
Base	1	1	2	3
Height	80	100	80	20

Drawing the histogram:

Horizontal (*x*-axis) axis (Time): Look at the intervals. The times go from 0 to 70. The smallest interval (Base 1) is 10 so go up in 10's.

Vertical (*y*-axis) axis (No. of customers): Always start at zero. The biggest number is the maximum height (i.e. 100).

No. of Customers
(Frequency)

Time (minutes)
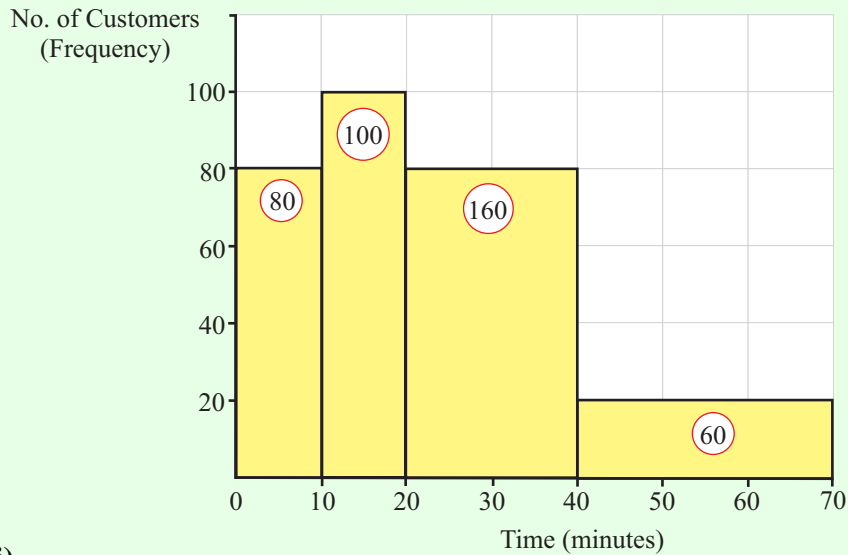
7 (b) (iii)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.

Ex. Class interval: $10 - 20$

Mid-interval value: $\dfrac{10+20}{2} = 15$

x	f	fx
5	80	400
15	100	1500
30	160	4800
55	60	3300
	400	10000

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + ... + f_N x_N}{f_1 + f_2 + ... + f_N} = \frac{\sum fx}{\sum f} \quad .... \; \textbf{2}$$

Mean time: $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{10000}{400} = 25$

7 (b) (iv)

160 customers were present in the time interval 20 – 30 minutes. All 160 customers could have been present for a time greater than 30 minutes.
Therefore, the greatest number of customers who could have been present for more than 30 minutes = 160 + 60 = 220.

7 (b) (v)

160 customers were present in the time interval 20 – 30 minutes. All 160 customers could have been present for a time less than 30 minutes.
Therefore, the least number of customers who could have been present for more than 30 minutes = 60.

2002

7 (c) The grouped frequency table below refers to the marks obtained by 85 students in a test:

Marks	0 – 40	40 – 55	55 – 70	70 – 100
Number of students	16	18	27	24

[Note: 40 – 55 means 40 marks or more but less than 55 etc.]

(i) What percentage of students obtained 55 marks or higher?

(ii) Name the interval in which the median lies.

(iii) Draw an accurate histogram to represent the data.

SOLUTION

7 (c) (i)

The number of students who received 55 marks or higher $= 27 + 24 = 51$

Therefore, percentage of student obtaining a mark of 55 or higher $= \dfrac{51}{85} \times 100\% = 60\%$

7 (c) (ii)

The median is the mark obtained by the middle student (the 42nd./43rd. student). This student lies in the 55 – 70 interval.

[34 students (16 + 18) have received less than 55 marks. 61 students (16 + 18 + 27) have received less than 70 marks. Therefore, the 42nd./43rd. student lies in the 55 – 70 interval.]

7 (c) (iii)

Marks	0 – 40	40 – 55	55 – 70	70 – 100
Number of students	16	18	27	24

Each column in the table is represented by a rectangular box. The area of the box corresponds to the frequency (no. of students).

Area (No. of students) = Base × Height = Frequency

Look at the marks. Look at the intervals. They are all divisible by 5. Make the interval 0 – 5 a base of one unit. Therefore the interval 0 – 40 has a base of 8 units. Divide the base into the area (frequency) to get the height of a box.

Draw a new table:

Interval (Marks)	0 – 40	40 – 55	55 – 70	70 – 100
Frequency (No. of students)	16	18	27	24
Base	8	3	3	6
Height	2	6	9	4

Drawing the histogram:

Horizontal (*x*-axis) axis (Marks): Look at the intervals. The marks go from 0 to 100. The smallest interval (Base 1) is 5.

Vertical (*y*-axis) axis (No. of students): Always start at zero. The biggest number is the maximum height (i.e. 9).

Number of Students (y-axis) vs Percentage Marks (x-axis). Bars labelled 16, 18, 27, 24.
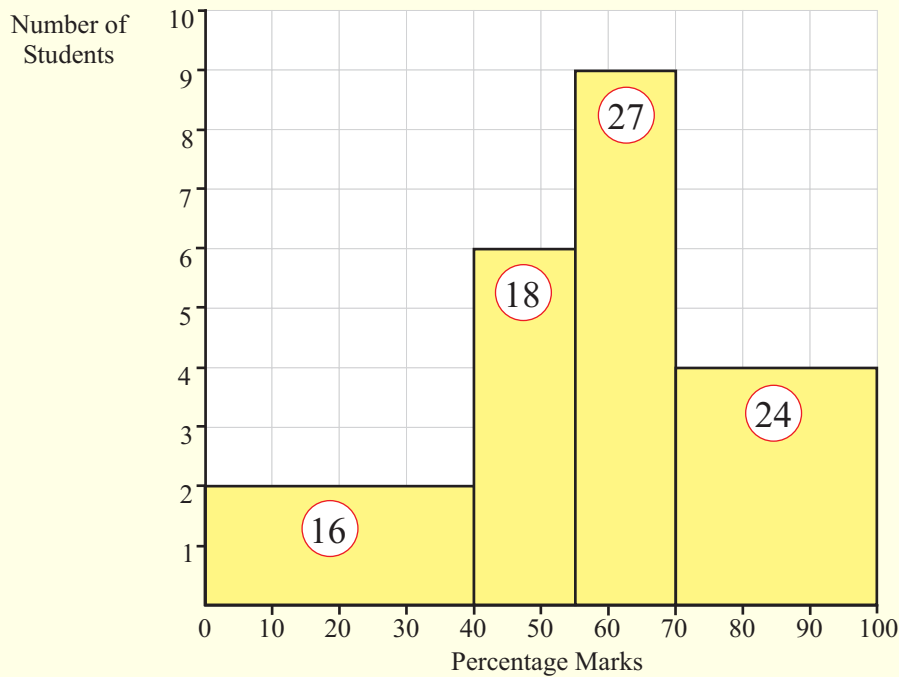
2000

7 (b) The table shows the distribution of points obtained by 50 people who took a driving test.

Points obtained	0 – 20	20 – 40	40 – 80	80 –100
Number of people	4	8	28	10

(i) Draw a histogram to illustrate the data.

(ii) To pass the driving test a person must obtain 65 points or more. What is the greatest possible number of people who passed the test?

SOLUTION

7 (b) (i)

Points obtained	0 – 20	20 – 40	40 – 80	80 – 100
Number of people	4	8	28	10

Each column in the table is represented by a rectangular box. The area of the box corresponds to the frequency (no. of students).

$$\boxed{\text{Area (No. of students)} = \text{Base} \times \text{Height} = \text{Frequency}}$$

Look at the points obtained. Pick out the smallest interval (0 – 20) and make this base one unit. Therefore the interval 40 – 80 has a base of 2 units. Divide the base into the area (frequency) to get the height of a box.
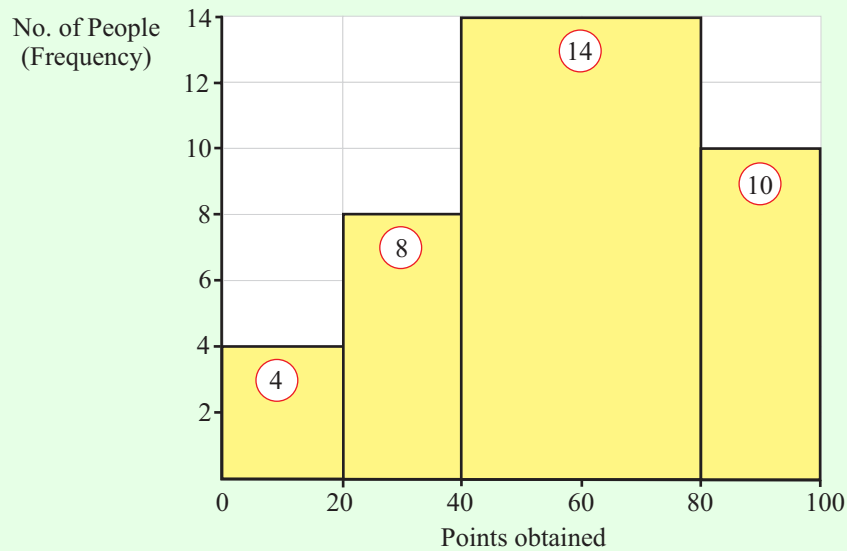
Draw a new table:

Interval (Points)	0 – 20	20 – 40	40 – 80	80 – 100
Frequency (No. of people)	4	8	28	10
Base	1	1	2	1
Height	4	8	14	10

Drawing the histogram:
Horizontal (*x*-axis) axis (Points): Look at the intervals. The points go from 0 to 100. The smallest interval (Base 1) is 20 so go up in 20's.
Vertical (*y*-axis) axis (No. of people): Always start at zero. The biggest number is the maximum height (i.e. 14).
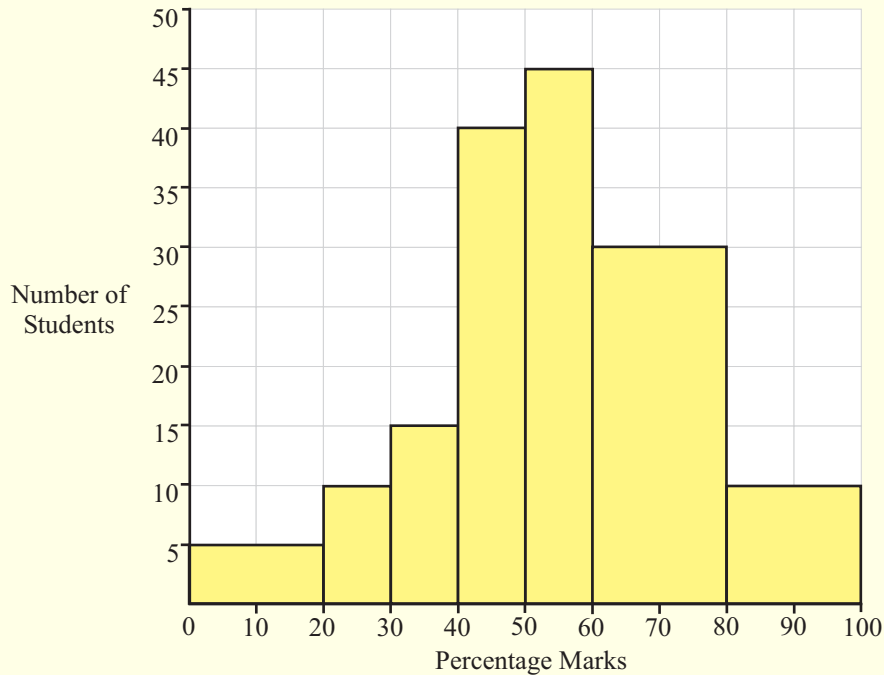

7 (b) (ii)

28 people received between 40 and 80 points. All 28 people in the group could have received 65 points or more. Therefore, the greatest possible number of people who passed the test = 28 + 10 = 38 people.

1998

7 (b) The distribution of percentage marks awarded to a group of 200 Leaving Certificate students in a particular subject is shown in the histogram below.

(i) If 45 students obtained between 50% and 60%, copy and complete the frequency table below.

Marks (%)	$0-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-80$	$80-100$
Frequency					45		

(ii) What is the greatest possible number of students who could have obtained a grade C or better (i.e. mark ≥ 55)?

SOLUTION

7 (b) (i)

Area (Frequency) = Base $\times$ Height

For the given interval: $45 = \text{Base} \times 45 \Rightarrow \text{Base} = 1$

As can be seen from the histogram, the given interval occupies 1 marked unit along the horizontal axis. Therefore, an interval of 20 has a base of 2 and an interval of 10 has a base of 1.

INTERVAL NO. 1: Area $= 2 \times 5 = 10$

INTERVAL NO. 2: Area $= 1 \times 10 = 10$

INTERVAL NO. 3: Area $= 1 \times 15 = 15$

INTERVAL NO. 4: Area $= 1 \times 40 = 40$

INTERVAL NO. 5: Given in the table

INTERVAL NO. 6: Area $= 2 \times 30 = 60$

INTERVAL NO. 7: Area $= 2 \times 10 = 20$

Now complete the table:

Marks (%)	$0-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-80$	$80-100$
Frequency	10	10	15	40	45	60	20

7 (b) (ii)

There are 45 students who obtained a mark between 50 and 60. It is possible that all 45 students in this group could have obtained 55 marks or more. Therefore, the greatest possible number of students who have obtained a grade C or better
$= 45 + 60 + 20 = 125$ students.

1996

7 (c) The grouped frequency table below shows the minutes spent in a shopping complex by a number of people:

Minutes	5 – 15	15 – 25	25 – 35	35 – 65
Number of people	10	50	80	60

Note that 5–15 means that 5 is included but 15 is not, etc.

(i)   Draw a histogram to illustrate the data.

(ii) Calculate the mean number of minutes spent per person in the shopping complex, taking 10, 20 etc. as mid-interval values.

SOLUTION

7 (c) (i)

Minutes	5 – 15	15 – 25	25 – 35	35 – 65
Number of people	10	50	80	60

Each column in the table is represented by a rectangular box. The area of the box corresponds to the frequency (no. of people).

$$\text{Area (No. of students)} = \text{Base} \times \text{Height} = \text{Frequency}$$

Look at the minutes. Pick out the smallest interval (5 – 15) and make this base one unit. Therefore the interval 35 – 65 has a base of 3 units. Divide the base into the area (frequency) to get the height of a box.

Draw a new table:

Interval (Minutes)	5 – 15	15 – 25	25 – 35	35 – 65
Number of people	10	50	80	60
Base	1	1	1	3
Height	10	50	80	20

Drawing the histogram:
Horizontal (*x*-axis) axis (Minutes): Look at the intervals. The minutes go from 5 to 65. The smallest interval (Base 1) is 10 so go up in 10's.
Vertical (*y*-axis) axis (No. of people): Always start at zero. The biggest number is the maximum height (i.e. 80).

CONT....

No. of People (Frequency)

Points obtained
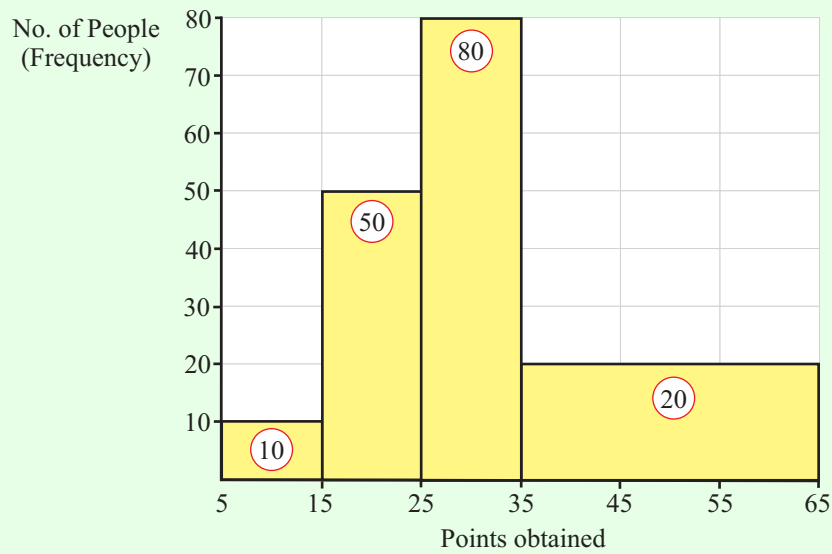
7 (c) (ii)

Draw up a frequency table using the mid-interval values. To get a mid-interval value add the two numbers together and divide by 2.

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_Nx_N}{f_1 + f_2 + \cdots + f_N} = \frac{\sum fx}{\sum f} \quad \cdots\cdots \; \mathbf{2}$$

Mean time: $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{6500}{200} = 32.5$ minutes

x	f	fx
10	10	100
20	50	1000
30	80	2400
50	60	3000
	200	6500