STATISTICS (Q 7, PAPER 2)

LESSON NO. 4: STANDARD DEVIATION

2006

7 (a) The mean of the five numbers 2, 4, 7, 8, 9 is 6.
Calculate the standard deviation of the five numbers, correct to one decimal place.

SOLUTION

STEPS
1. Find the mean.
2. Draw up a table of x, d and d^2.
3. Apply the standard deviation formula.

1. This is done for you.

$\bar{x} = 6$

2. The deviation, d, is given by the formula:
$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.
To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
2	–4	16
4	–2	4
7	1	1
8	2	4
9	3	9
		34

3. $\sigma = \sqrt{\dfrac{34}{5}} = 2.6$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}} \quad \ldots\ldots\ 4$$

2005

7 (b) There are fourteen questions in an examination.
The table below shows the performance of the candidates.

Correct responses	0 – 2	3 – 5	6 – 8	9 – 11	12 – 14
Number of candidates	1	2	6	8	3

(i) Using mid-interval values, calculate the mean number of correct responses.

(ii) Calculate the standard deviation, correct to one decimal place.

SOLUTION

If you are asked to find the mean and standard deviation of a frequency distribution, set it out in a table as shown.

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_Nx_N}{f_1 + f_2 + \ldots\ldots\ldots + f_N} = \frac{\sum fx}{\sum f} \quad \ldots\ldots\ 2$$

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}} \quad \ldots\ldots\ 5$$

x	f	fx	d	d^2	fd^2
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
	$\sum f$	$\sum fx$			$\sum fd^2$

Work out the mean first. Then work out d using $d = (x - \bar{x})$. Finally, calculate the standard deviation.

CONT....

x	f	fx	d	d^2	fd^2
1	1	1	– 7·5	56·25	56·25
4	2	8	– 4·5	20·25	40·5
7	6	42	– 1·5	2·25	13·5
10	8	80	1·5	2·25	18·0
13	3	39	4·5	20·25	60·75
	20	170			189

Draw up a table in the way as shown on the left. The mid-interval values, x, are obtained by adding the class interval values together and dividing by two.

(i) $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{170}{20} = 8 \cdot 5$

(ii) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f}} = \sqrt{\dfrac{189}{20}} = 3 \cdot 1$

2004

7 (a) The mean of the set of numbers {1, 3, 7, 9} is 5.
Find the standard deviation, correct to one decimal place.

Solution

Steps
1. Find the mean.
2. Draw up a table of x, d and d^2.
3. Apply the standard deviation formula.

1. This is done for you.

$\bar{x} = 5$

2. The deviation, d, is given by the formula:
$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.
To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
1	–4	16
3	–2	4
7	2	4
9	4	16
		40

3. $\sigma = \sqrt{\dfrac{40}{4}} = \sqrt{10} = 3.2$

$\sigma = \sqrt{\dfrac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\dfrac{\sum d^2}{N}}$

.... 4

2003

7 (b) (i) The mean of the following five numbers is 10. Find the standard deviation of the numbers.

7, 9, 10, 11, 13.

(ii) The mean of the following five numbers is also 10. Find the standard deviation of these numbers.

5, 7, 9, 13, 16.

(iii) What does comparing the two standard deviations tell you about the two sets of numbers?

SOLUTION

7 (b) (i)

STEPS
1. Find the mean.
2. Draw up a table of x, d and d^2.
3. Apply the standard deviation formula.

1. This is done for you.

$\bar{x} = 10$

2.

The deviation, d, is given by the formula:
$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.
To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
7	–3	9
9	–1	1
10	0	0
11	1	1
13	3	9
		20

3. $\sigma = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$

.... 4

7 (b) (ii)

1. This is done for you.

$\bar{x} = 10$

2.

The deviation, d, is given by the formula:
$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.
To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
5	–5	25
7	–3	9
9	–1	1
13	3	9
16	6	36
		80

3. $\sigma = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$

.... 4

7 (b) (iii)

The standard deviation, σ, is a measure of the spread of the values about the mean.

There is a greater spread of numbers about the mean with the second set of numbers.

2001

7 (a) (i) Calculate the mean of the following numbers

2, 3, 5, 7, 8.

(ii) Hence, calculate the standard deviation of the numbers correct to one decimal place.

SOLUTION

7 (a) (i)

The MEAN or AVERAGE of a set of numbers is calculated by adding the numbers together and dividing by the number of numbers.

$$\text{Mean} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

The mean is denoted by $\bar{x}$.

$$\bar{x} = \frac{2+3+5+7+8}{5}$$

$$\Rightarrow \bar{x} = \frac{25}{5} = 5$$

$$\bar{x} = \frac{x_1 + x_2 + \dots\dots\dots\dots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$ 1

7 (a) (ii)

STEPS
1. Find the mean.
2. Draw up a table of x, d and d^2.
3. Apply the standard deviation formula.

1. This is done in part **(i)**.

$\bar{x} = 5$

2.

The deviation, d, is given by the formula:

$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.

To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
2	–3	9
3	–2	4
5	0	0
7	2	4
8	3	9
		26

3. $\sigma = \sqrt{\frac{26}{5}} = 2.3$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ 4

1999

7 (c) The number of minutes taken by 20 pupils to answer a short question is shown in the following distribution table:

Minutes	2 – 4	4 – 6	6 – 8	8 – 10
Number of pupils	6	9	4	1

By taking the data at mid-interval values, calculate

(i) the mean number of minutes taken per pupil

(ii) the standard deviation, correct to one place of decimals.

SOLUTION

If you are asked to find the mean and standard deviation of a frequency distribution, set it out in a table as shown.

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_Nx_N}{f_1 + f_2 + \ldots\ldots\ldots + f_N} = \frac{\sum fx}{\sum f} \quad \ldots\ldots \; 2$$

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}} \quad \ldots\ldots \; 5$$

x	f	fx	d	d^2	fd^2
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
	$\sum f$	$\sum fx$			$\sum fd^2$

Work out the mean first. Then work out d using $d = (x - \overline{x})$. Finally, calculate the standard deviation.

x	f	fx	d	d^2	fd^2
3	6	18	– 2	4	24
5	9	45	0	0	0
7	4	28	2	4	16
9	1	9	4	16	16
	20	100			56

Draw up a table in the way as shown on the left. The mid-interval values, x, are obtained by adding the class interval values together and dividing by two.

(i) $\overline{x} = \frac{\sum fx}{\sum f} = \frac{100}{20} = 5$

(ii) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f}} = \sqrt{\frac{56}{20}} = 1\cdot 7$

1998

7 (c) The following table shows the sizes, in hectares, of 20 farms in a particular area:

No. of hectares	15 – 45	45 – 75	75 – 105	105 – 195
Number of farms	1	4	8	7

By taking the data at mid-interval values, calculate

(i) the mean number of hectares per farm

(ii) the standard deviation, correct to the nearest hectare.

SOLUTION

If you are asked to find the mean and standard deviation of a frequency distribution, set it out in a table as shown.

$$\boxed{\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_Nx_N}{f_1 + f_2 + \ldots\ldots + f_N} = \frac{\sum fx}{\sum f}} \ \ldots\ldots \ 2$$

$$\boxed{\sigma = \sqrt{\frac{\sum fd^2}{\sum f}}} \ \ldots\ldots \ 5$$

x	f	fx	d	d^2	fd^2
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
	$\sum f$	$\sum fx$			$\sum fd^2$

Work out the mean first. Then work out d using $d = (x - \bar{x})$. Finally, calculate the standard deviation.

x	f	fx	d	d^2	fd^2
30	1	30	– 72	5184	5184
60	4	240	– 42	1764	7056
90	8	720	– 12	144	1152
150	7	1050	48	2304	16128
	20	2040			29520

Draw up a table in the way as shown on the left. The mid-interval values, x, are obtained by adding the class interval values together and dividing by two.

(i) $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{2040}{20} = 102$

(ii) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f}} = \sqrt{\dfrac{29520}{20}} = 38$

1997

7 (b) $\{2, 5, 6, 4.5, 2.5\}$

Show that 4 is the mean of this set of numbers.

Then, calculate the standard deviation, correct to one place of decimals.

SOLUTION

STEPS
1. Find the mean.
2. Draw up a table of x, d and d^2.
3. Apply the standard deviation formula.

1.

The **MEAN** or **AVERAGE** of a set of numbers is calculated by adding the numbers together and dividing by the number of numbers.

$$\text{Mean} = \frac{\text{Sum of the numbers}}{\text{Number of numbers}}$$

The mean is denoted by $\bar{x}$.

$$\bar{x} = \frac{2+5+6+4.5+2.5}{5}$$

$$\Rightarrow \bar{x} = \frac{20}{5} = 4$$

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots\ldots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$ 1

2.

The deviation, d, is given by the formula:

$d = (x - \bar{x}) = (\text{Number} - \text{Mean})$.

To work out d, get the difference between each number, x, and the mean, $\bar{x}$.

x	d	d^2
2	–2	4
5	1	1
6	2	4
4.5	0.5	0.25
2.5	–1.5	2.25
		11.5

3. $\sigma = \sqrt{\dfrac{11.5}{5}} = 1.5$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ 4