COUNTING & PROBABILITY (Q 6, PAPER 2)

LESSON NO. 6: PROBABILITY RULES

2001

6 (a) Sarah and Jim celebrate their birthdays in a particular week (Monday to Sunday inclusive).
Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that

(i) Sarah's birthday is on Friday

(ii) Sarah's birthday and Jim's birthday are both on Friday?

SOLUTION

6 (a) (i)

There are 7 days in the week.

p(Sarah's birthday is on Friday) $= \frac{1}{7}$

6 (a) (ii)

$p(A \text{ and then } B) = p(A) \times p(B)$ 5

p(Sarah's birthday is on Friday) $= \frac{1}{7}$

p(Jim's birthday is on Friday) $= \frac{1}{7}$

p(Sarah's and Jim's birthday is on Friday) $= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

1999

6 (c) Twelve blood samples are tested in a laboratory. Of these it is found that five blood samples are of type A, four of type B and the remaining three are of type O.
Two blood samples are selected at random from the twelve.
What is the probability that

(i) the two samples are of type A

(ii) one sample is of type B and the other sample is of type O

(iii) the two samples are of the same blood type?

SOLUTION

6 (c) (i)

A	**A**	**A**	**A**
A	**B**	**B**	**B**
B	**O**	**O**	**O**

$p(\mathbf{A} \text{ and } \mathbf{A}) = p(\mathbf{A}) \times p(\mathbf{A})$

$p(A \text{ and then } B) = p(A) \times p(B)$ 5

First pick: $p(\mathbf{A}) = \dfrac{\text{No. of } \mathbf{A}\text{'s}}{\text{No. of samples}} = \dfrac{5}{12}$

For the second pick, there are 4 **A**'s left out of 11 samples.

Second pick: $p(\mathbf{A}) = \dfrac{\text{No. of } \mathbf{A}\text{'s}}{\text{No. of samples}} = \dfrac{4}{11}$

$p(\mathbf{A} \text{ and } \mathbf{A}) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

CONT.....

6 (c) (ii)

$p(\mathbf{B} \text{ and } \mathbf{O}) = p(\mathbf{B}) \times p(\mathbf{O}) \times 2$

You need to multiply your answer by two because you could pick **B** first and **O** second or **O** first and **B** second.

First pick: $p(\mathbf{B}) = \dfrac{\text{No. of } \mathbf{B}\text{'s}}{\text{No. of samples}} = \dfrac{4}{12} = \dfrac{1}{3}$

For the second pick there are eleven samples left to pick from.

Second pick: $p(\mathbf{O}) = \dfrac{\text{No. of } \mathbf{O}\text{'s}}{\text{No. of samples}} = \dfrac{3}{11}$

$p(\mathbf{B} \text{ and } \mathbf{O}) = \frac{1}{3} \times \frac{3}{11} \times 2 = \frac{2}{11}$

6 (c) (iii)

$p(\text{Same sample}) = p(2\ \mathbf{A}\text{'s}) \text{ OR } p(2\ \mathbf{B}\text{'s}) \text{ OR } p(2\ \mathbf{O}\text{'s})$

[OR means add the probabilities together.]

$p(\text{Same sample}) = p(2\ \mathbf{A}\text{'s}) + p(2\ \mathbf{B}\text{'s}) + p(2\ \mathbf{O}\text{'s})$

$p(\mathbf{A} \text{ and } \mathbf{A}) = \frac{5}{33}$

$p(\mathbf{B} \text{ and } \mathbf{B}) = \frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$

$p(\mathbf{O} \text{ and } \mathbf{O}) = \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

$p(\text{Same sample}) = \frac{5}{33} + \frac{1}{11} + \frac{1}{22} = \frac{19}{66}$

1997

6 (c) Two people are chosen at random from a large crowd. Each person names the day of the week on which he or she was born. Assuming that each day is equally likely, what is the probability that

(i) both people were born on a Friday

(ii) one person was born on a Tuesday and the other was born on a Thursday

(iii) the two people were born on different days?

SOLUTION

6 (c) (i)

There are 7 days in the week. Call the two people A and B.

$p(A \text{ and then } B) = p(A) \times p(B)$ 5

$p(\text{A has her birthday on Friday}) = \frac{1}{7}$

$p(\text{B has her birthday on Friday}) = \frac{1}{7}$

$p(\text{Birthdays are both on Friday}) = \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

CONT.....

6 (c) (ii)

The probability that A's birthday is on a Tuesday is $\frac{1}{7}$. The probability that B's birthday is on a Thursday is $\frac{1}{7}$. However, it could be the other way round so multiply your answer by two.

p(Birthdays on Tuesday and Thursday) $= \frac{1}{7} \times \frac{1}{7} \times 2 = \frac{2}{49}$

6 (c) (iii)

A has her birthday on any day. The probability of A having her birthday on any day is $\frac{7}{7} = 1$.

The probability that B has her birthday on a different day to A is $\frac{6}{7}$.

p(Birthdays on different days) $= 1 \times \frac{6}{7} = \frac{6}{7}$

1996

6 (a) A bag contains 24 beads of which 12 are red, 8 are blue and 4 are white.
A bead is taken at random from the bag.
What is the probability that the colour of the bead is
(i) blue
(ii) red or white?

SOLUTION

5 (a) (i)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \; 4$$

$$p(\text{Blue}) = \frac{\text{No. of blue discs}}{\text{No. of discs}} = \frac{8}{24} = \frac{1}{3}$$

5 (a) (ii)

$$p(\text{Red or White}) = \frac{\text{No. of Red and White discs}}{\text{No. of discs}} = \frac{16}{24} = \frac{2}{3}$$