COUNTING & PROBABILITY (Q 6, PAPER 2)

LESSON NO. 5: SPECIAL PROBABILITY PROBLEMS

2005

6 (c) Seven horses run in a race.
All horses finish the race and no two horses finish the race at the same time.

(i) In how many different orders can the seven horses finish the race?

(ii) A person is asked to predict the correct order of the first three horses to finish the race. How many different such predictions can be made?

(iii) A person is asked to predict, in any order, the first three horses to finish the race. How many different such predictions can be made?

(iv) A person selects two of the seven horses at random. What is the probability that the selected horses are the first two horses to finish the race?

SOLUTION

6 (c) (i)

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of ways in which you can arrange 7 different objects all taken, no repeats = 7!

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

6 (c) (ii)

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

You are asked the number of ways in which you can pick 3 horses from 7 horses where order is important.

${}^7P_3 = 7 \times 6 \times 5 = 210$

6 (c) (iii)

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

You are asked the number of ways in which you can select 3 horses from 7 horses where order is not important.

${}^7C_3 = \binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$

CONT....

6 (c) (iv)

How many ways can 2 horses be picked selected from 7 horses where order is not important?

$${}^{7}C_{2} = \binom{7}{2} = \frac{7 \times 6}{2 \times 1} = 21$$

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \; 4$$

As there are 21 possibilities for selecting 2 horses out of 7 horses, there is a 21 to 1 chance of these 2 horses finishing in the first two places.

p(2 particular horses are the first two in the race) $= \frac{1}{21}$

2003

6 (c) In a certain school the examination subjects for senior students are grouped as follows:

Compulsory Subjects	Block A	Block B	Block C
Irish English mathematics	French German	biology home economics construction studies accounting	business organisation history physics

As well as taking all three of the compulsory subjects, each student must choose *one* subject from Block A, *two* from Block B and *one* from Block C.

(i) In choosing two subjects from Block B, how many different selections are possible?

(ii) In choosing the full range of subjects, how many different selections are possible?

(iii) One student has already decided to do German and construction studies. How many different selections of the remaining subjects are possible for this student?

(iv) If the student referred to in part **(iii)** selects her remaining subjects at random, what is the probability that she will select both biology and physics?

SOLUTION

6 (c) (i)

Compulsory Subjects	Block A	Block B	Block C
Irish English mathematics	French German	biology home economics construction studies accounting	business organisation history physics
[*Must take all 3*]	[*Must choose 1 from 2*]	[*Must choose 2 from 4*]	[*Must choose 1 from 2*]

CONT....

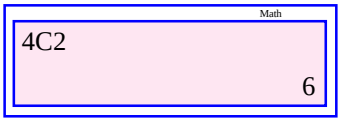
How many ways (order is **not** important) can 2 subjects be selected from 4 subjects?

$${}^4C_2 = \frac{4\times 3}{2\times 1} = 6$$

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

CALCULATOR: Calculate ${}^4C_2$.

4 SHIFT nCr 2 =



6 (c) (ii)

Compulsory subjects: You need to select 3 from 3 (no choice) **AND**
Block A: You need to select 1 from 2 **AND**
Block B: You need to select 2 from 4 **AND**
Block C: You need to select 1 from 3.
NOTE: **AND** means multiply.

No. of different selections $= {}^3C_3 \times {}^2C_1 \times {}^4C_2 \times {}^3C_1 = 1\times 2\times 6\times 3 = 36$

6 (c) (iii)

Compulsory Subjects	Block A	Block B	Block C
Irish English mathematics	French German	biology home economics construction studies accounting	business organisation history physics
[*Choice made*]	[*Choice made*]	[*Must choose 1 from 3*]	[*Must choose 1 from 2*]

No. of different selections $= 1\times 1\times {}^3C_1 \times {}^3C_1 = 1\times 1\times 3\times 3 = 9$

6 (c) (iv)

There are 9 possible selections. Selecting biology and physics is one such selection.

p(Physics and Biology) $= \frac{1}{9}$

2002

6 (b) A meeting is attended by 23 men and 21 women.
Of the men, 14 are married and the others are single.
Of the women, 8 are married and the others are single.

(i) A person is picked at random. What is the probability that the person is a woman?

(ii) A person is picked at random. What is the probability that the person is married?

(iii) A man is picked at random. What is the probability that he is married?

(iv) A woman is picked at random. What is the probability that she is single?

SOLUTION

6 (b)

Draw up a table containing all the information.

	Married	Single
Men (23)	14	7
Women (21)	8	13

Total number of people: 44
Total number of married people: 22
Total number of single people: 20

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

6 (b) (i)

$$p(\text{Woman}) = \frac{\text{No. of women}}{\text{No. of people}} = \frac{21}{44}$$

6 (b) (ii)

$$p(\text{Married person}) = \frac{\text{No. of married people}}{\text{No. of people}} = \frac{22}{44} = \frac{1}{2}$$

6 (b) (iii)

$$p(\text{Married man}) = \frac{\text{No. of married men}}{\text{No. of men}} = \frac{14}{23}$$

6 (b) (iv)

$$p(\text{Single woman}) = \frac{\text{No. of single women}}{\text{No. of women}} = \frac{13}{21}$$

2000

6 (b) In a class, there are 15 boys and 13 girls. Four boys wear glasses and three girls wear glasses.
A pupil is picked at random from the class.
(i) What is the probability that the pupil is a boy?

(ii) What is the probability that the pupil wears glasses?

(iii) What is the probability that the pupil is a boy who wears glasses?

A girl is picked at random from the class.
(iv) What is the probability that she wears glasses?

SOLUTION

6 (b)

Draw up a table containing all the information.

	Wear Glasses	Does not wear glasses
Boys (15)	4	11
Girls (13)	3	10

Total number of pupils: 28
Total number of pupils who wear glasses: 7
Total number of pupils who do not wear glasses: 21

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \; 4$$

6 (b) (i)

$$p(\text{Boy}) = \frac{\text{No. of boys}}{\text{No. of pupils}} = \frac{15}{28}$$

6 (b) (ii)

$$p(\text{Pupil who wears glasses}) = \frac{\text{No. of pupils who wear glasses}}{\text{No. of pupils}} = \frac{7}{28} = \frac{1}{4}$$

6 (b) (iii)

$$p(\text{Boy who wears glasses}) = \frac{\text{No. of boys who wear glasses}}{\text{No. of pupils}} = \frac{4}{28} = \frac{1}{7}$$

6 (b) (iv)

$$p(\text{Girl who wears glasses}) = \frac{\text{No. of girls who wear glasses}}{\text{No. of girls}} = \frac{3}{13}$$