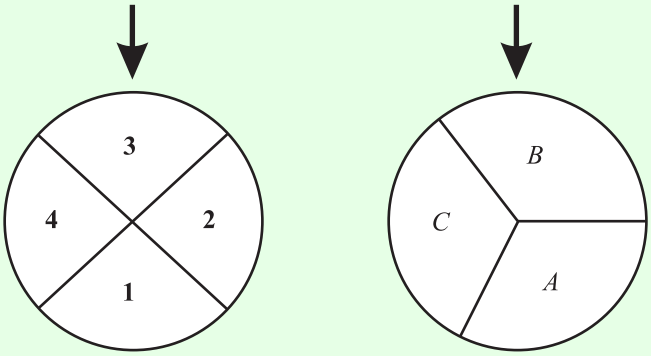
Counting & Probability (Q 6, Paper 2)

Lesson No. 4: Probability and the List Method

2007

6 (b) The diagram shows two wheels.



The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.
The second wheel is divided into three equal segments labelled *A*, *B* and *C*.
A game consists of spinning the two wheels and noting the segments that stop at the arrows. For example, the outcome shown is (3, *B*).

(i) Write down all the possible outcomes.

(ii) What is the probability that the outcome is (2, *C*)?

(iii) What is the probability that the outcome is an odd number with the letter *A*?

(iv) What is the probability that the outcome includes the letter *C*?

Solution

6 (b) (i)

(1, *a*), (1, *b*), (1, *c*), (2, *a*), (2, *b*), (2, *c*), (3, *a*), (3, *b*), (3, *c*), (4, *a*), (4, *b*), (4, *c*)

$$\boxed{p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}} \;\ldots\ldots\; 4$$

6 (b) (ii)

(1, *a*), (1, *b*), (1, *c*), (2, *a*), (2, *b*), (2, *c*), (3, *a*), (3, *b*), (3, *c*), (4, *a*), (4, *b*), (4, *c*)

$$p((2, C)) = \frac{1}{12}$$

6 (b) (iii)

(1, *a*), (1, *b*), (1, *c*), (2, *a*), (2, *b*), (2, *c*), (3, *a*), (3, *b*), (3, *c*), (4, *a*), (4, *b*), (4, *c*)

$$p(\text{Odd No.} + \text{'}A\text{'}) = \frac{2}{12} = \frac{1}{6}$$

6 (b) (iv)

(1, *a*), (1, *b*), (1, *c*), (2, *a*), (2, *b*), (2, *c*), (3, *a*), (3, *b*), (3, *c*), (4, *a*), (4, *b*), (4, *c*)

$$p(\text{No.} + \text{'}C\text{'}) = \frac{4}{12} = \frac{1}{3}$$

2006

6 (c) Three coins are tossed. Each coin gives either a head or a tail.

(i) Write down all the possible outcomes. For example, "H, T, H" or "head, tail, head" is one possible outcome.

(ii) Find the probability that the result is three tails.

(iii) Find the probability that the result includes no more than one head.

(iv) Find the probability that the result has at least one head.

SOLUTION

6 (c) (i)

There are 8 possible outcomes:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

6 (c) (ii)

$$\boxed{p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}} \text{ ...... } 4$$

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$$p(3 \text{ Tails}) = \frac{1}{8}$$

6 (c) (iii)

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

No more than one head means one head or no heads are present.

$$p(\text{No more than one head}) = \frac{4}{8} = \frac{1}{2}$$

6 (c) (iv)

At least one head means you can have 1 head, 2 heads or 3 heads.

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$$p(\text{At least one head}) = \frac{7}{8}$$

2005

6 (b) Ten teams take part in a competition. The teams are divided into two groups. Teams A, B, C, D and E are in group 1 and teams U, V, X, Y and Z are in group 2.
In the final, the winning team from group 1 plays the winning team from group 2.
Each team is equally likely to win its group.

(i) How many different team pairings are possible for the final?

(ii) What is the probability that team C plays team X in the final?

(iii) What is the probability that team A plays in the final?

(iv) What is the probability that team B does not play in the final?

SOLUTION

6 (b) (i)

Group 1	Group 2
A	U
B	V
C	X
D	Y
E	Z

Make a list:
{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}
There are 25 possible pairings.

6 (b) (ii)

$$\boxed{p(E)=\frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}} \quad \ldots\ldots \quad 4$$

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{C plays X})=\frac{1}{25}$

6 (b) (iii)

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{A plays in final})=\frac{5}{25}=\frac{1}{5}$

6 (b) (iv)

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{B does not play in final})=\frac{20}{25}=\frac{4}{5}$

2004

6 (c) Four cards, numbered 2, 3, 4, 5 respectively, are shuffled and then placed in a row with the numbers visible.
Find the probability that
(i) the numbers shown are in the order: 5, 4, 3, 2

(ii) the first and second numbers are both even

(iii) the sum of the two middle numbers is 7.

SOLUTION

6 (c) (i)

You can do this question the long way by writing out all the possibilities or the shorter way by using some formulae.

2	3	4	5	3	2	4	5	4	3	2	5	5	3	4	2
2	3	5	4	3	2	5	4	4	3	5	2	5	3	2	4
2	4	5	3	3	4	5	2	4	2	5	3	5	4	2	3
2	4	3	5	3	4	2	5	4	2	3	5	5	4	3	2
2	5	4	3	3	5	4	2	4	5	2	3	5	2	4	3
2	5	3	4	3	5	2	4	4	5	3	2	5	2	3	4

Long way: There are 24 possibilities. The is one possibility with the order as 5, 4, 3, 2.

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \quad 4$$

$p(5,4,3,2) = \frac{1}{24}$

Short way:

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

How many ways can you arrange 4 different objects, all taken, no repeats (order is important)?

${}^4P_3 = 4 \times 3 \times 2 \times 1 = 24$

5, 4, 3, 2 is one such arrangement.

$\therefore\ {}^4P_3 = 4 \times 3 \times 2 \times 1 = 24$

CONT....

6 (c) (ii)

Long way:

2 3 4 5	3 2 4 5	4 3 2 5	5 3 4 2
2 3 5 4	3 2 5 4	4 3 5 2	5 3 2 4
2 4 5 3	3 4 5 2	4 2 5 3	5 4 2 3
2 4 3 5	3 4 2 5	4 2 3 5	5 4 3 2
2 5 4 3	3 5 4 2	4 5 2 3	5 2 4 3
2 5 3 4	3 5 2 4	4 5 3 2	5 2 3 4

As you can see there are 4 arrangements out of 24 arrangments where the first and second numbers are even.

$p(\text{First 2 numbers are even}) = \frac{4}{24} = \frac{1}{6}$

Short way:
Use the multiplication principle.
There are two even numbers. The first box must be even so there are 2 ways to fill the first box.
The second box must also be even. There is only one way to fill the second box once the first box is filled.
There are two ways to fill the third box as there are only two numbers left once the first two are filled.
Finally there is one way to fill the last box.

Number of ways = $2 \times 1 \times 2 \times 1 = 4$

Must be even

6 (c) (iii)

The best way to do this is by listing all the possibilities.

2 3 4 5	3 2 4 5	4 3 2 5	5 3 4 2
2 3 5 4	3 2 5 4	4 3 5 2	5 3 2 4
2 4 5 3	3 4 5 2	4 2 5 3	5 4 2 3
2 4 3 5	3 4 2 5	4 2 3 5	5 4 3 2
2 5 4 3	3 5 4 2	4 5 2 3	5 2 4 3
2 5 3 4	3 5 2 4	4 5 3 2	5 2 3 4

As you can see there are eight possible arrangements where the two middle numbers add up to 7.

$p(\text{Sum of middle two numbers is 7}) = \frac{8}{24} = \frac{1}{3}$

2003

6 (b) Two women, Ann and Bríd, and two men, Con and Declan, sit in a row for a photograph.

(i) How many different arrangements of the four people are possible?

(ii) Write out the four possible arrangements that have the two women in the middle.

(iii) If the arrangement of the four people is chosen at random from all of the possible arrangements, what is the probability that the two women will be in the middle?

SOLUTION

6 (b) (i)

You can do this question the long way by writing out all the possibilities or the shorter way by using some formulae.

Long way:

A B C D	B A C D	C B A D	D B C A
A B D C	B A D C	C B D A	D B A C
A C B D	B C A D	C A B D	D C B A
A C D B	B C D A	C A D B	D C A B
A D B C	B D A C	C D B A	D A B C
A D C B	B D C A	C D A B	D A C B

You can see there are 24 arrangements of 4 people.

Short way:

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

How many ways can you arrange 4 different objects, all taken, no repeats (order is important)?

$${}^4P_3 = 4\times3\times2\times1 = 24$$

4 (b) (ii)

(Con, Ann, Brid, Declan), (Declan, Ann, Brid, Con),
(Con, Brid, Ann, Declan), (Declan, Brid, Ann, Con)

4 (b) (iii)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

$$p(\text{2 women in the middle}) = \frac{4}{24} = \frac{1}{6}$$

2000

6 (a) To go to work, a woman can walk or travel by bus or travel by car with a neighbour. To return home, she can walk or travel by bus.

(i) In how many different ways can the woman go to and return from work on any one day?

(ii) List all of these different ways.

SOLUTION

6 (a) (i)

There are 3 ways to woman can go to work **AND** 2 ways she can return home.

Number of ways the woman can go to and return from work $= 3 \times 2 = 6$

NOTE: **AND** means multiply.

6 (a) (ii)

{(Walk, Walk), (Walk, Bus), (Bus, Walk), (Bus, Bus), (Car, Walk), (Car, Bus)}