COUNTING & PROBABILITY (Q 6, PAPER 2)

LESSON NO. 2: PERMUTATIONS

2007

6 (c) (i) How many different three-digit numbers can be formed from the digits 2, 3, 4, 5, 6, if each of the digits can be used only once in each number?

(ii) How many of the numbers are less than 400?

(iii) How many of the numbers are divisible by 5?

(iv) How many of the numbers are less than 400 and divisible by 5?

SOLUTION

6 (c) (i)

The number of arrangements of *n* different objects taking *r* at a time with no repeats $= {}^nP_r$ **2**

The number of arrangements of five different numbers taking three at a time with no repeats $= {}^5P_3 = 5 \times 4 \times 3 = 60$

CALCULATOR: Calculate ${}^5P_3$.

5 SHIFT **nPr** **3** **=**

Math
5P3
60

OR

Use the multiplication principle:

The 1st. box can be filled by any one of 5 digits.
AND then the 2nd. box can only be filled by one of 4 digits, given that the 1st. box is already filled by a digit.
AND then the 3rd. box can be only filled by one of 3 digits.
Now multiply the three numbers together to get 60 arrangements.

Number of ways = 5 × 4 × 3 = 60

6 (c) (ii)

The first box can only be filled 2 ways, by either a 2 or 3. Any other digits will give a number greater than 400. Once this is filled, there are 4 ways to fill the second box and 3 ways to fill the third box.

Number of ways = 2 × 4 × 3 = 24

Must be a 2 or 3

6 (c) (iii)

The last digit must be the 5 in order to be divisible by five. Therefore, there is only 1 way to fill the last box. This means there are 4 ways to fill the first box and 3 ways to fill the second box.

Number of ways = 4 × 3 × 1 = 12

5

CONT.....

6 (c) (iv)

The first box must be filled by a 2 or 3 (two ways) in order to have a number less than 400.
The last box must be filled by a 5 (one way) in order to be divisible by 5.
This means there are 3 ways to fill the second box.

Number of ways = $2 \times 3 \times 1 = 6$

[ ] [ ] [5]

Must be a 2 or 3

2006

6 (b) Niamh uses a password formed from one letter of her name followed by four of the digits from 1 to 9. She does not use any digit more than once.

(i) How many such passwords can be formed?

(ii) How many of the passwords begin with N?

(iii) How many of the passwords end in an even digit?

(iv) How many of the passwords begin with N and use only odd digits?

SOLUTION

6 (b) (i)

There are 5 letters in her name. Therefore, there are 5 ways to fill the first box.
There are nine digits so there are 9 ways to fill the second box. As there are no repeats there are 8 ways to fill the third box and so on.

Number of ways = $5 \times 9 \times 8 \times 7 \times 6 = 15{,}120$

[ ] [ ] [ ] [ ] [ ]

6 (b) (ii)

The first box is filled with N (one way). The rest of the boxes are filled in the same way.

Number of ways = $1 \times 9 \times 8 \times 7 \times 6 = 3{,}024$

[N] [ ] [ ] [ ] [ ]

6 (b) (iii)

There are 5 ways to fill the first box (from the letters of NIAMH).
Now fill the last box with the restriction. This box must contain an even digit. There are 4 ways to fill it (with a 2, 4, 6 or 8).
Once the last box is filled, there are 8 ways to fill the second box and so on.

Number of ways = $5 \times 8 \times 7 \times 6 \times 4 = 6{,}720$

[ ] [ ] [ ] [ ] [ ]

Must be even

CONT.....

6 (b) (iv)

There is one way to fill the first box (with an N).
There are 5 ways to fill the second box as there are 5 odd digits (1, 3, 5, 7 or 9).
There are 4 ways to fill the third box and 3 ways to fill the last box.

Number of ways = $1 \times 5 \times 4 \times 3 \times 2 = 120$

N				

2005

6 (a) (i) Evaluate 6!

SOLUTION

6! means start with 6 and then multiply by 5 and then 4 all the way down to 1.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

CALCULATOR: Calculate 6!

6 SHIFT *x*! =

Math
6!
720

2004

6 (a) The letters of the word CUSTOMER are arranged at random.

(i) How many different arrangements are possible?

(ii) How many of these arrangements begin with the letter C?

SOLUTION

6 (a) (i)

The number of arrangements of *n* different objects all taken, no repeats = *n*! 3

The number of arrangements of 8 different letters all taken, no repeats = 8!

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$

CALCULATOR: Calculate 8!

8 SHIFT *x*! =

Math
8!
40,320

OR

There are 8 ways to fill the first box. Once this is filled, there are 7 ways to fill the second box and so on.

Number of ways = $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$

6 (a) (ii)

There is only one way to fill the first box (with the letter C). Once this is filled, there are 7 ways to fill the second box and so on.

Number of ways = $1 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$

C							

2002

6 (c) The digits 0, 1, 2, 3, 4, 5 are used to form four-digit codes. A code cannot begin with 0 and no digit is repeated in any code.

(i) Write down the largest possible four-digit code.

(ii) Write down the smallest possible four-digit code.

(iii) How many four-digit codes can be formed?

(iv) How many of the four-digit codes are greater than 4000?

SOLUTION

6 (c) (i)

5 digits: 0, 1, 2, 3, 4, 5

0 cannot be in the first position.

No repeats.

Largest possible number: **5** **4** **3** **2**

6 (c) (ii)

Smallest possible number: **1** **0** **2** **3**

6 (c) (iii)

Number of 4 digit codes:

Number of ways = 5 × 5 × 4 × 3 = 300

Cannot be a zero —— (first box)

Can be a zero but not what is in the first box (second box)

6 (c) (iv)

Number of 4 digit codes greater than 4000:

The first box must be filled with a 4 or 5 but not a zero (2 ways).

The second box can be filled 5 ways, the third 4 ways and so on.

Number of ways = 2 × 5 × 4 × 3 = 120

Must be a 4 or 5 —— (first box)

2001

6 (b) (i) How many different arrangements can be made using all the letters of the word IRELAND?

(ii) How many arrangements begin with the letter I?

(iii) How many arrangements end with the word LAND?

(iv) How many begin with I and end with LAND?

SOLUTION

6 (b) (i)

There are 7 different letters in the word IRELAND.

MULTIPLICATION PRINCIPLE:

There are 7 ways to fill the first box. Once this box is filled, there are 6 ways to fill the second box and so on.

Number of ways = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 7 different letters all taken, no repeats = 7!

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$

CALCULATOR: Calculate 7!

7 SHIFT $x!$ =

Math

7!

5,040

6 (b) (ii)

Number of ways = 1 × 6 × 5 × 4 × 3 × 2 × 1 = 720

I						

There is only one way to fill the first box (with an **I**).
Once this is filled, there are 6 ways to fill the second box and so on.

6 (b) (iii)

Number of ways = 3 × 2 × 1 × 1 × 1 × 1 × 1 = 6

			L	**A**	**N**	**D**

There is only one way to fill the last four boxes.
Once these are filled, there are 3 ways to fill the first box, 2 ways to fill the second box and one way to fill the third box.

6 (b) (iv)

Number of ways = 1 × 2 × 1 × 1 × 1 × 1 × 1 = 2

I			**L**	**A**	**N**	**D**

There is one way to fill the first box and the last four boxes. This means there are 2 letters left to fill the second box. Once this is filled, there is only one letter left to fill the third box.

2000

6 (c) (i) How many different five-digit numbers can be formed from the digits 2, 3, 4, 5, 6? Each digit can be used once only in each number.

(ii) How many of the numbers are even?

(iii) How many of the numbers are less than 40 000?

(iv) How many of the numbers are both even and less than 40 000?

SOLUTION

6 (c) (i)

Multiplication Principle:
There are 5 digits in total. Therefore, there are 5 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = 5 × 4 × 3 × 2 × 1 = 120

OR

The number of arrangements of *n* different objects all taken, no repeats = *n*! 3

The number of arrangements of 5 different numbers all taken, no repeats = 5!

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

CALCULATOR: Calculate 5!

5 SHIFT *x*! =

5!
120

6 (c) (ii)

Always work on the box with the restriction first. The last box must be filled with an even digit. There are 3 ways to fill this box (with a 2, 4 or 6).
Once this is filled, there are 4 ways to fill the first box and so on.

Number of ways = 4 × 3 × 2 × 1 × 3 = 72

Must be an even digit

6 (c) (iii)

Always work on the box with the restriction first. The first box must be filled with a 2 or 3 (two ways).
Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = 2 × 4 × 3 × 2 × 1 = 48

Must be a 2 or 3

6 (c) (iv)

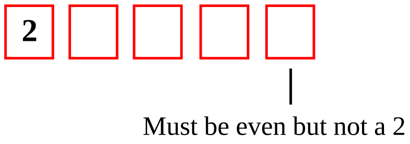
This can be a little tricky because the 2 is needed to make the number even and also to the make the number less than 40,000. Therefore, consider both possibilities for filling the first box.

Fill the first box with a 2. There is one way to fill it.

The last box must contain an even digit. There are two even digits left (a 4 or 6). Therefore, there are two ways to fill it.

Once these two boxes are filled, there are three ways to fill the second box and so on.

Number of ways $= 1 \times 3 \times 2 \times 1 \times 2 = 12$



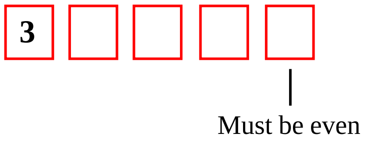
OR

Fill the first box with a 3. There is one way to fill it.

The last box must contain an even digit. There are three even digits (a 2, 4 or 6). Therefore, there are three ways to fill it.

Once these two boxes are filled, there are three ways to fill the second box and so on.

Number of ways $= 1 \times 3 \times 2 \times 1 \times 3 = 18$



OR means add. Add the two possibilities together.

Number of numbers less than 40,000 and even = 12 + 18 = 30

1999

6 (b) (i) In how many different ways can the 5 letters of the word ANGLE be arranged?

(ii) How many of these arrangements begin with a vowel?

(iii) In how many of the arrangements do the two vowels come together?

SOLUTION

6 (b) (i)

Multiplication Principle:
There are 5 letters in total. Therefore, there are 5 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = $5 \times 4 \times 3 \times 2 \times 1 = 120$

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 5 different letters all taken, no repeats = 5!

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

CALCULATOR: Calculate 5!

5 SHIFT $x!$ =

Math
5!
120

6 (b) (ii)

There are 2 vowels (A and E). There are 2 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = $2 \times 4 \times 3 \times 2 \times 1 = 48$

Must be a vowel

6 (b) (iii)

Glue the two vowels together and treat as a single unit.

One such arrangement **AE** **N** **G** **L**

Must be a vowel

There are 4! ways of arranging 4 objects **AND** then there are 2! ways of arranging the two objects glued together.

No. of arrangements of the 5 letters with the vowels side by side $= 4! \times 2! = 24 \times 2 = 48$

NOTE: The word **AND** means multiply.

1998

6 (c) (i) How many different numbers, each with 3 digits or less, can be formed from the digits 1, 2, 3, 4, 5? Each digit can be used only once in each number.

(ii) How many of the above numbers are even?

SOLUTION

6 (c) (i)

There are five digits (1, 2, 3, 4 and 5). How many three digit numbers, two digits numbers and single digit numbers can be formed from these five digits with no repeats?

THREE DIGIT NUMBERS:

There are 5 ways to fill the first box. Once this is filled there are 4 ways to fill the second box and three ways to fill the third box. Therefore, there are 60 possible three digit numbers.

Number of ways = $5 \times 4 \times 3 = 60$

TWO DIGIT NUMBERS:

There are 5 ways to fill the first box. Once this is filled there are 4 ways to fill the second box. Therefore, there are 20 possible two digit numbers.

Number of ways = $5 \times 4 = 20$

ONE DIGIT NUMBERS:

Obviously, there are 5 one digit numbers.

The total number of numbers of 3 digits or less: $60 + 20 + 5 = 85$

6 (c) (ii)

EVEN THREE DIGIT NUMBERS:

The last box must be filled with an even digit in order for the number to be even. There are two ways to fill the last box (with a 2 or 4). Once the last box is filled there are fours ways to fill the first box and three ways to fill the second box.

Number of ways = $4 \times 3 \times 2 = 24$

Must be even

EVEN TWO DIGIT NUMBERS:

The last box must be filled with an even digit in order for the number to be even. There are two ways to fill the last box (with a 2 or 4). Once the last box is filled there are fours ways to fill the first box.

Number of ways = $4 \times 2 = 8$

Must be even

EVEN ONE DIGIT NUMBERS:

Obviously there are 2 even one digit numbers.

The total number of even numbers of 3 digits or less: $24 + 8 + 2 = 34$

1997

6 (b) (i) In how many different ways can the letters of the word CARPET be arranged?

(ii) How many of these arrangements begin with A?

(iii) In how many of the arrangements do the two vowels come together?

SOLUTION

6 (b) (i)

USE THE MULTIPLICATION PRINCIPLE: There are 6 ways to fill the first box. Once this is filled, there are 5 ways to fill the second box and so on.

Number of ways = 6 × 5 × 4 × 3 × 2 × 1 = 720

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 6 different letters all taken, no repeats = 6!

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

CALCULATOR: Calculate 6!

6 SHIFT $x!$ =

Math
6!
720

6 (b) (ii)

Number of ways = 1 × 5 × 4 × 3 × 2 × 1 = 120

A					

There is only one way to fill the first box (with the letter **A**). Once this box is filled there are only 5 ways to fill the second box. Once this box is filled there are only 4 ways to fill the third box and so on.

6 (b) (iii)

Glue the two vowels together and treat as a single unit.

One such arrangement

AE	C	R	P	T

There are 5! ways of arranging 5 objects **AND** then there are 2! ways of arranging the two objects glued together.

No. of arrangements of the 5 letters with the vowels side by side $= 5! \times 2! = 120 \times 2 = 240$

NOTE: The word **AND** means multiply.

1996

6 (b) There are 5 horses, *A*, *B*, *C*, *D* and *E*, in a race. Each horse takes a different time to complete the race. On completing the race,

(i) in how many different placing arrangements can the 5 horses finish?

(ii) if *A* is placed first and *B* last, in how many different placing arrangements can the other horses finish?

SOLUTION

5 (b) (i)

MULTIPLICATION PRINCIPLE: There are 5 horses that can finish in first place. Once a horse finishes in first place there are 4 horses that can finish in second place and so on.

Number of ways = 5 × 4 × 3 × 2 × 1 = 120

1st	2nd	3rd	4th	5th

OR

The number of arrangements of *n* different objects all taken, no repeats = *n*! 3

The number of arrangements of 5 different horses all taken, no repeats = 5!

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

CALCULATOR: Calculate 5!

5 SHIFT x! =

Math
5!
120

5 (b) (ii)

Number of ways = 1 × 3 × 2 × 1 × 1 = 6

1st	2nd	3rd	4th	5th
A				**B**

There is one way to fill the first box (with **A**) and one way to fill the last box (with **B**). Once these are filled there are 3 horses left to fill the second box, 2 left to fill the third box and 1 horse left to fill the fourth box.