Counting & Probability (Q 6, Paper 2)

Lesson No. 1: Combinations

2006

6 (a) Evaluate $5\binom{8}{3}-4\binom{8}{4}$.

Solution

$$5\binom{8}{3}-4\binom{8}{4}=5\left(\frac{8\times7\times6}{3\times2\times1}\right)-4\left(\frac{8\times7\times6\times5}{4\times3\times2\times1}\right)$$

$$=5(56)-4(70)$$

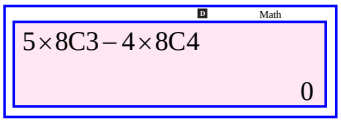
$$=280-280=0$$

Calculator: Calculate $5\binom{8}{3}-4\binom{8}{4}$.

5 × 8 SHIFT nCr

3 − 4 × 8

SHIFT nCr 4 =



$$\binom{12}{3}=\frac{12\times11\times10}{3\times2\times1}=220$$

2005

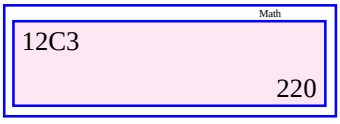
6 (a) (ii) Evaluate $\binom{12}{3}$.

Solution

$$\binom{12}{3}=\frac{12\times11\times10}{3\times2\times1}=220$$

Calculator: Calculate $\binom{12}{3}$.

12 SHIFT nCr 3 =



2004

6 (b) A committee of 3 people is selected from a group of 15 doctors and 12 dentists. In how many different ways can the 3 people be selected

(i) if there are no restrictions

(ii) if the selection must contain exactly 2 doctors

(iii) if the selection must contain at least 1 doctor and at least 1 dentist

(iv) if the selection must contain one specific doctor and one specific dentist?

SOLUTION

6 (b) (i)

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

The number of ways of selecting 3 people from 27 people is: ${}^{27}C_3 = \binom{27}{3} = \frac{27\times26\times25}{3\times2\times1} = 2,925$

15 Doctors 12 Dentists
↓
3 Places

CALCULATOR: Calculate ${}^{27}C_3$.

27 SHIFT nCr 3 =

Math
27C3
2,925

6 (b) (ii)

You need to select 2 doctors from 15 doctors **AND** 1 dentist from 12 dentists.

15 Doctors 12 Dentists
↓ ↓
2 Places 1 Place

${}^{15}C_2 \times {}^{12}C_1 = \left(\frac{15\times14}{2\times1}\right)\times\left(\frac{12}{1}\right) = 1,260$

NOTE: **AND** means you multiply.

6 (b) (iii)

At least one doctor and at least one dentist can mean 2 doctors and 1 dentist **OR** 1 doctor and 2 dentists.

15 Doctors 12 Dentists
↓ ↓
2 Places 1 Place

2 doctors and 1 dentist: ${}^{15}C_2 \times {}^{12}C_1 = 1,260$

OR

1 dentist and 2 doctors: ${}^{15}C_1 \times {}^{12}C_2 = \left(\frac{15}{1}\right)\times\left(\frac{12\times11}{2\times1}\right) = 990$

15 Doctors 12 Dentists
↓ ↓
1 Place 2 Places

NOTE: **OR** means you add.

2 doctors and 1 dentist **OR** 1 dentist and 2 doctors = 1,260 + 990 = 2,250

6 (b) (iv)

If one specific doctor is chosen and one specific dentist is chosen you are left to pick one person from 14 doctors and 11 dentists (25 people).

14 Doctors 11 Dentists
↓
1 Place

${}^{25}C_1 = \left(\frac{25}{1}\right) = 25$

2002

6 (a) There are eight questions on an examination paper.

(i) In how many different ways can a candidate select six questions?

(ii) In how many different ways can a candidate select six questions if one particular question must always be selected?

SOLUTION

6 (a) (i)

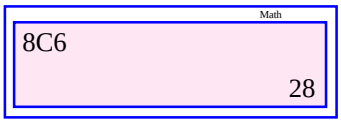
The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

In how many ways can you select 6 questions out of 8 questions (order is not important)?

$${}^8C_6 = \binom{8}{6} = \frac{8\times 7\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}}{\cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{3}\times 2\times 1} = \frac{8\times 7}{2\times 1} = 28 \quad [\textbf{NOTE: } \binom{8}{6} = \binom{8}{2}.]$$

CALCULATOR: Calculate ${}^8C_6$.

8 SHIFT nCr 6 =



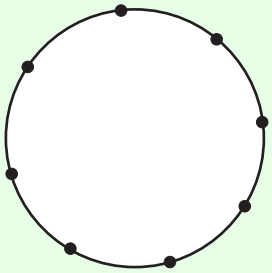
6 (a) (ii)

If one question must be answered, this means you have to select 5 questions from 7 questions.

$${}^7C_5 = \binom{7}{5} = \binom{7}{2} = \frac{7\times 6}{2\times 1} = 21$$

2001

6 (c) (i) Eight points lie on a circle, as in the diagram. How many different lines can be drawn by joining any two of the eight points?



(ii) Find the value of the natural number n such that

$$\binom{n}{2} = 105.$$

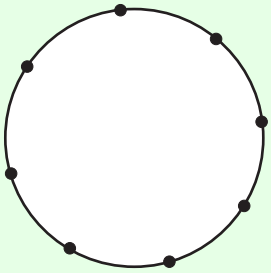
[Note: $\binom{n}{2}$ may also be written as ${}^{n}C_2$.]

SOLUTION

6 (c) (i)

The number of selections of n different objects taking r at a time $= {}^{n}C_r = \binom{n}{r}$ **1**

LINES are formed by selecting points in pairs. There are 8 points. In how many ways can you select 2 points from 8 different points?



$${}^{8}C_2 = \frac{8\times 7}{2\times 1} = 28$$

CALCULATOR: Calculate ${}^{8}C_2$.

8 SHIFT **nCr** **2** **=**

Math
8C2
28

6 (c) (ii)

$$\binom{n}{2} = {}^{n}C_2 = 105$$

$$\Rightarrow \frac{n(n-1)}{2\times 1} = 105 \text{ [Multiply across by 2.]}$$

$$\Rightarrow n^2 - n = 210$$

$$\Rightarrow n^2 - n - 210 = 0$$

$$\Rightarrow (n-15)(n+14) = 0$$

$\therefore n = 15$ [Ignore the negative solution as n is a natural number (a whole positive number).]

1999

6 (a) (i) In how many ways can a team of 5 players be chosen from a panel of 8 players?

(ii) If a certain player must be on the team, in how many ways can the team be then chosen.

SOLUTION

6 (a) (i)

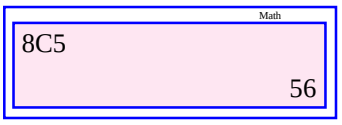
The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

The number of selections of 8 different players taking 5 at a time $= {}^8C_5 = \binom{8}{5}$.

$${}^8C_5 = \binom{8}{5} = \frac{8\times7\times6\times\cancel{5}\times\cancel{4}}{\cancel{5}\times\cancel{4}\times3\times2\times1} = 56$$

CALCULATOR: Calculate ${}^8C_5$.

8 SHIFT **nCr** **5** **=**



6 (a) (ii)

If a certain player must be on the team, you need to choose 4 players from the remaining 7 players.

$${}^7C_4 = \binom{7}{4} = \frac{7\times6\times5\times\cancel{4}}{\cancel{4}\times3\times2\times1} = 35$$

1998

6 (b) A committee of 4 people is to be formed from a group of 7 men and 6 women.

(i) How many different committees can be formed?

(ii) On how many of these committees is there an equal number of men and of women?

SOLUTION

6 (b) (i)

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

In total there are 13 people (7 men and 6 women).
In how many ways can you pick 4 people on the committee from 13 people?

7 Men 6 Women
↓
4 Places

$${}^{13}C_4 = \binom{13}{4} = \frac{13\times12\times11\times10}{4\times3\times2\times1} = 715$$

CALCULATOR: Calculate ${}^{13}C_4$.

13 SHIFT nCr 4 =

Math
13C4
714

6 (b) (ii)

You need to pick 2 men and 2 women for the committee to have an equal number of men and women.
How many ways can you pick 2 men from 7 men **AND** 2 women from 6 women?

Note: **AND** means multiply.

7 Men → 2 Men
6 Women → 2 Women

$${}^7C_2 \times {}^6C_2 = \binom{7}{2}\times\binom{6}{2} = \left(\frac{7\times6}{2\times1}\right)\times\left(\frac{6\times5}{2\times1}\right) = 21\times15 = 315$$

1997

6 (a) A class of 29 students wins a prize. Two members of the class are chosen to receive the prize. How many different pairs of students can be chosen?

SOLUTION

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

In how many ways can 2 students be selected from 29 students (order is not important)?

$${}^{29}C_2 = \binom{29}{2} = \frac{29\times28}{2\times1} = 406$$

CALCULATOR: Calculate ${}^{29}C_2$.

29 SHIFT nCr 2 =

Math
29C2
406