COUNTING & PROBABILITY (Q 6, PAPER 2)

2011

6. **(a)** **(i)** Find 4!

(ii) Simplify $\frac{6(5!)}{5(4!)}$.

(b) The letters in the word FERMAT are arranged taking all of the letters each time.
How many different arrangements are possible if

(i) there are no restrictions

(ii) the arrangements begin with the letter F

(iii) the arrangements begin with the letter F and end with a vowel

(iv) the two vowels are together?

(c) The table below shows how the students in a school usually travel to school.

	Walk	Cycle	Other
Boys	157	123	166
Girls	184	91	172

(i) A student is picked at random.
What is the probability that the student is a boy?

(ii) A student is picked at random.
What is the probability that the student walks to school?

(iii) A boy is picked at random.
What is the probability that he cycles to school?

(iv) A girl is picked at random.
What is the probability that she does not walk to school?

SOLUTION

6 (a) (i)

$4! = 4 \times 3 \times 2 \times 1 = 24$

6 (a) (ii)

$$\frac{6(5!)}{5(4!)} = \frac{6(5\times4\times3\times2\times1)}{5(4\times3\times2\times1)} = \frac{6\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} = 6$$

6 (b) (i)

The number of arrangements of n different objects all taken, no repeats = $n!$

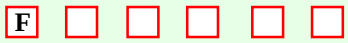
FERMAT

There are 6 ways to fill the first box. Once this is filled there are 5 ways to fill the second box. Once this is filled there are 4 ways to fill the third box and so on.

Number of ways = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

6 (b) (ii)

There is one way to fill the first box, with the letter F. This means there are 5 ways to fill the second box.
Once this is filled there are 4 ways to fill the third box and so on.



Number of ways = $1 \times 5 \times 4 \times 3 \times 2 \times 1 = 120$

6 (b) (iii)

There are two possibilities for finishing with a vowel. Do each one separately and add the two answers together.

Number of ways = $1 \times 4 \times 3 \times 2 \times 1 \times 1 = 24$

Number of ways = $1 \times 4 \times 3 \times 2 \times 1 \times 1 = 24$

$\oplus$

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6 (b) (iv)

One such arrangement

There are 5! ways of arranging 5 objects AND then there are 2! ways of arranging the two objects glued together.

No. of arrangements of the 6 letters with the vowels side by side $= 5! \times 2! = 120 \times 2 = 240$

NOTE: The word AND means multiply.

6 (c) (i)

	Walk	Cycle	Other
Boys	157	123	166
Girls	184	91	172

Number of boys = 157 + 123 + 166 = 446
Number of girls = 184 + 91 + 172 = 447
Number of students = 446 + 447 = 893

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$

$$p(\text{Boy}) = \frac{\text{Number of boys}}{\text{Number of students}} = \frac{446}{893}$$

6 (c) (ii)

Number of students who walk to school = 157 + 184 = 341

$$p(\text{Student walking}) = \frac{\text{Number of students walking}}{\text{Number of students}} = \frac{341}{893}$$

6 (c) (iii)

A boy is picked at random. You are asked to find the probability that he cycles to school.

$$p(\text{Boy cycles}) = \frac{\text{Number of boys cycling}}{\text{Number of boys}} = \frac{123}{446}$$

6 (c) (iv)

A girl is picked at random. You are asked to find the probability that she does not walk to school.

Number of girls not walking to school = 91 + 172 = 263

$$p(\text{Girl not walking}) = \frac{\text{Number of girls not walking}}{\text{Number of girls}} = \frac{263}{447}$$