COUNTING & PROBABILITY (Q 6, PAPER 2)

2010

6 (a) (i) In how many different ways can a committee of four people be selected from ten people?

(ii) If one particular person must be on the committee, in how many different ways can the committee be selected?

(b) Tickets for a raffle are placed in a box.
The box contains 15 blue tickets and 10 red tickets.
Tickets are drawn at random from the box and they are not replaced.
What is the probability that

(i) the first ticket drawn is red

(ii) the first ticket drawn and the second ticket drawn are both red

(iii) the first ticket drawn is red and the second ticket drawn is blue

(iv) the first two tickets drawn are different in colour?

(c) A code consists of a four-digit number which is formed from the digits 3 to 9 inclusive.
No digit can occur more than once in the code.

(i) Write down the smallest possible four-digit code.

(ii) How many different codes are possible?

(iii) How many of the four-digit codes are greater than 6000?

(iv) How many of the four-digit codes are divisible by 2?

SOLUTION

6 (a) (i)

$${}^{10}C_4 = \binom{10}{4} = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

10 people
↓
4 Places

The number of selections of n different objects taking r at a time $= {}^{n}C_r = \binom{n}{r}$

6 (a) (ii)

If one particular person must be on the committee, there are 3 places to fill from 9 people.

$${}^{9}C_3 = \binom{9}{3} = \frac{9\times8\times7}{3\times2\times1} = 84$$

9 people
↓
3 Places

6 (b) (i)

15 blue tickets, 10 red tickets,
25 tickets
There are 10 red tickets to pick out from 25 tickets.

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$

$$p(\text{Red}) = \frac{10}{25} = \frac{2}{5}$$

6 (b) (ii)

On the first pick, there are 10 reds to pick from 25 tickets.
On the second pick, there are 9 reds to pick from 24 tickets.

$$p(A \text{ and then } B) = p(A) \times p(B)$$

$$p(\text{Red and then Red}) = \frac{10}{25} \times \frac{9}{24} = \frac{3}{20}$$

6 (b) (iii)

On the first pick, there are 10 reds to pick from 25 tickets.
On the second pick, there are 15 blues to pick from 24 tickets.

$$p(\text{Red and then Blue}) = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$$

6 (b) (iv)

The probability that the 2 tickets drawn are different colours could mean that the first ticket drawn is red and the second blue or the first is blue and the second red. Simply multiply the previous answer by 2 to take into account both possibilities.

$$p(\text{Red and Blue}) = \frac{1}{4} \times 2 = \frac{1}{2}$$

6 (c) (i)

Seven digits: 3, 4, 5, 6, 7, 8, 9 [No repeats]
Smallest four digit number: 3456

6 (c) (ii)

$$7 \times 6 \times 5 \times 4 = 840$$

There are 7 ways to fill the first digit. Once this is filled, there are 6 ways to fill the second digit and so on.

6 (c) (iii)

$$4 \times 6 \times 5 \times 4 = 480$$

To be greater than 6,000, the first digit must be 6, 7, 8 or 9. There are 4 ways to fill the first digit. Once this is filled, there are 6 ways to fill the second digit and so on.

6 (c) (iv)

$$\square \times \square \times \square \times \square = 360$$

To be divisible by 2 the last digit must be even. There are 3 ways (8, 6 or 4) to fill the last digit. Once this is filled, there are 6 ways to fill the first digit and so on.