Counting & Probability (Q 6, Paper 2)

2009

6 (a) (i) Evaluate $\binom{7}{2}$.

(ii) Evaluate $\binom{7}{2}+\binom{7}{5}$.

(b) There are 210 boys and 240 girls in a school. The school has a junior cycle and a senior cycle. The number of boys and the number of girls in each cycle is shown in the table.

	Boys	Girls
Junior Cycle	120	130
Senior Cycle	90	110

(i) A student is picked at random.
What is the probability that the student is a boy?

(ii) A student is picked at random.
What is the probability that the student is in the senior cycle?

(iii) A junior cycle student is picked at random.
What is the probability that the student is a girl?

(iv) A boy is picked at random.
What is the probability that he is in the senior cycle?

(c) Three boys and two girls are seated in a row as a group.
In how many different ways can the group be seated if

(i) there are no restrictions on the order of seating

(ii) there must be a boy at the beginning of the row

(iii) there must be a boy at the beginning of the row and a boy at the end of the row

(iv) the two girls must be seated beside each other?

Solution

6 (a) (i)

$$\binom{7}{2}=\frac{7\times 6}{2\times 1}=21$$

6 (a) (ii)

$$\binom{7}{5}=\frac{7\times 6\times 5\times 4\times 3}{5\times 4\times 3\times 2\times 1}=21$$

$$\binom{7}{2}+\binom{7}{5}=21+21=42$$

6 (b) (i)

No. of boys = 210
No. of students = 450

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$

$$p(\text{Boy}) = \frac{\text{No. of boys}}{\text{No. of students}} = \frac{210}{450} = \frac{7}{15}$$

6 (b) (ii)

No. of senior cycle students = 200
No. of students = 450

$$p(\text{Senior cycle student}) = \frac{\text{No. of senior cycle students}}{\text{No. of students}} = \frac{200}{450} = \frac{4}{9}$$

6 (b) (iii)

No. of girls in the junior cycle = 130
No. of junior cycle students = 250

$$p(\text{Junior cycle girl}) = \frac{\text{No. of girls in the junior cycle}}{\text{No. of junior cycle students}} = \frac{130}{250} = \frac{13}{25}$$

6 (b) (iv)

No. of boys in the junior cycle = 90
No. of boys = 210

$$p(\text{Senior cycle boy}) = \frac{\text{No. of senior cycle boys}}{\text{No. of boys}} = \frac{90}{210} = \frac{3}{7}$$

6 (c) (i)

$5 \times 4 \times 3 \times 2 \times 1 = 120$

There are 5 ways to fill the first seat.
Once this seat is filled, there are 4 ways to fill the second seat.
Once the first 2 seats are filled, there are 3 ways to fill the third seat and so on.

6 (c) (ii)

$3 \times 4 \times 3 \times 2 \times 1 = 72$

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There are 3 ways to fill the first seat given a boy must sit there.
Once the first seat is filled, there are 4 ways to fill the second seat and so on.

6 (c) (iii)

$3 \times 3 \times 2 \times 1 \times 2 = 36$

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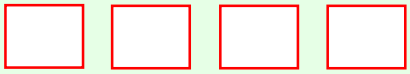
There are 3 ways to fill the first seat given a boy must sit there.
Once the first seat is filled, there are 2 ways to fill the last seat as a boy must sit there.
Once these 2 seats are filled, there are 3 ways to fill the second seat and so on.

6 (c) (iv) $4 \times 3 \times 2 \times 1 \times 2 = 48$



Glue the 2 girls together and treat this combination as a single seat. Therefore, there are now 4 seats.
There are 4 ways to fill the first seat (with one of 3 boys or the girl/girl combination).
Once this seat is filled, there are 3 ways to fill the second seat and so on.
The 2 girls can also switch places which doubles the number of combinations. So multiply by 2 to get the final answer.