COUNTING & PROBABILITY (Q 6, PAPER 2)

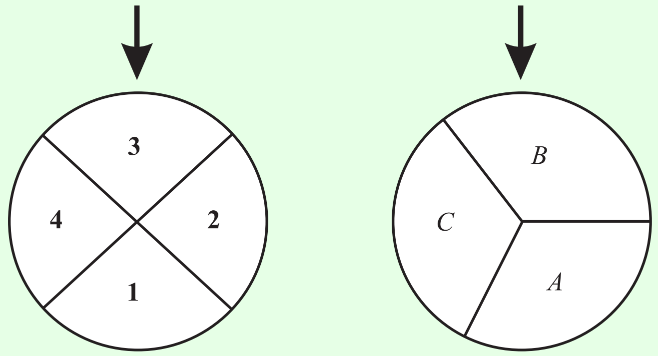
2007

6 (a) One letter is chosen at random from the letters of the word EUCLID.

(i) Find the probability that the letter chosen is D.

(ii) Find the probability that the letter chosen is a vowel.

(b) The diagram shows two wheels.



The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.
The second wheel is divided into three equal segments labelled *A*, *B* and *C*.
A game consists of spinning the two wheels and noting the segments that stop at the arrows. For example, the outcome shown is (3, *B*).

(i) Write down all the possible outcomes.

(ii) What is the probability that the outcome is (2, *C*)?

(iii) What is the probability that the outcome is an odd number with the letter *A*?

(iv) What is the probability that the outcome includes the letter *C*?

(c) (i) How many different three-digit numbers can be formed from the digits 2, 3, 4, 5, 6, if each of the digits can be used only once in each number?

(ii) How many of the numbers are less than 400?

(iii) How many of the numbers are divisible by 5?

(iv) How many of the numbers are less than 400 and divisible by 5?

SOLUTION

6 (a) (i)

$$p(\mathbf{D})=\frac{\text{No. of } \mathbf{D}\text{'s}}{\text{No. of letters}}=\frac{1}{6}$$

$$p(E)=\frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

6 (a) (ii)

$$p(\mathbf{Vowel})=\frac{\text{No. of vowels}}{\text{No. of letters}}=\frac{2}{6}=\frac{1}{3}$$

6 (b) (i)

(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

6 (b) (ii)

(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)

$$p((2, C)) = \frac{1}{12}$$

6 (b) (iii)

(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)

$$p(\text{Odd No.} + \text{'}A\text{'}) = \frac{2}{12} = \frac{1}{6}$$

6 (b) (iv)

(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c), (4, a), (4, b), (4, c)

$$p(\text{No.} + \text{'}C\text{'}) = \frac{4}{12} = \frac{1}{3}$$

6 (c) (i)

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

The number of arrangements of five different numbers taking three at a time with no repeats $= {}^5P_3 = 5 \times 4 \times 3 = 60$

CALCULATOR: Calculate ${}^5P_3$.

5 SHIFT nPr 3 =

Math

5P3

60

OR

Use the multiplication principle:

The 1st. box can be filled by any one of 5 digits.
AND then the 2nd. box can only be filled by one of 4 digits, given that the 1st. box is already filled by a digit.
AND then the 3rd. box can be only filled by one of 3 digits.
Now multiply the three numbers together to get 60 arrangements.

Number of ways = 5 × 4 × 3 = 60

6 (c) (ii)

The first box can only be filled 2 ways, by either a 2 or 3. Any other digits will give a number greater than 400. Once this is filled, there are 4 ways to fill the second box and 3 ways to fill the third box.

Number of ways = $2 \times 4 \times 3 = 24$

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Must be a 2 or 3

6 (c) (iii)

The last digit must be the 5 in order to be divisible by five. Therefore, there is only 1 way to fill the last box. This means there are 4 ways to fill the first box and 3 ways to fill the second box.

Number of ways = $4 \times 3 \times 1 = 12$

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6 (c) (iv)

The first box must be filled by a 2 or 3 (two ways) in order to have a number less than 400. The last box must be filled by a 5 (one way) in order to be divisible by 5.
This means there are 3 ways to fill the second box.

Number of ways = $2 \times 3 \times 1 = 6$

$\boxed{}\ \boxed{}\ \boxed{5}$

Must be a 2 or 3