

COUNTING & PROBABILITY (Q 6, PAPER 2)

2006

- 6 (a) Evaluate $5\binom{8}{3} - 4\binom{8}{4}$.
- (b) Niamh uses a password formed from one letter of her name followed by four of the digits from 1 to 9. She does not use any digit more than once.
- (i) How many such passwords can be formed?
 - (ii) How many of the passwords begin with N?
 - (iii) How many of the passwords end in an even digit?
 - (iv) How many of the passwords begin with N and use only odd digits?
- (c) Three coins are tossed. Each coin gives either a head or a tail.
- (i) Write down all the possible outcomes. For example, “H, T, H” or “head, tail, head” is one possible outcome.
 - (ii) Find the probability that the result is three tails.
 - (iii) Find the probability that the result includes no more than one head.
 - (iv) Find the probability that the result has at least one head.

SOLUTION

6 (a)

$$\begin{aligned} 5\binom{8}{3} - 4\binom{8}{4} &= 5\left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1}\right) - 4\left(\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}\right) \\ &= 5(56) - 4(70) \\ &= 280 - 280 = 0 \end{aligned}$$

CALCULATOR: Calculate $5\binom{8}{3} - 4\binom{8}{4}$.

5	×	8	SHIFT	nCr	5 × 8C3 - 4 × 8C4 0
3	-	4	×	8	
SHIFT	nCr	4	=		

6 (b) (i)

There are 5 letters in her name. Therefore, there are 5 ways to fill the first box. There are nine digits so there are 9 ways to fill the second box. As there are no repeats there are 8 ways to fill the third box and so on.

$$\text{Number of ways} = 5 \times 9 \times 8 \times 7 \times 6 = 15,120$$

6 (b) (ii)

The first box is filled with N (one way). The rest of the boxes are filled in the same way.

$$\text{Number of ways} = 1 \times 9 \times 8 \times 7 \times 6 = 3,024$$

6 (b) (iii)

There are 5 ways to fill the first box (from the letters of NIAMH). Now fill the last box with the restriction. This box must contain an even digit. There are 4 ways to fill it (with a 2, 4, 6 or 8). Once the last box is filled, there are 8 ways to fill the second box and so on.

$$\text{Number of ways} = 5 \times 8 \times 7 \times 6 \times 4 = 6,720$$

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Must be even

6 (b) (iv)

There is one way to fill the first box (with an N). There are 5 ways to fill the second box as there are 5 odd digits (1, 3, 5, 7 or 9). There are 4 ways to fill the third box and 3 ways to fill the last box.

$$\text{Number of ways} = 1 \times 5 \times 4 \times 3 \times 2 = 120$$

6 (c) (i)

There are 8 possible outcomes:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

6 (c) (ii)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \dots\dots \textcircled{4}$$

HHH, HHT, HTH, THH, HTT, THT, TTH, **TTT**

$$p(3 \text{ Tails}) = \frac{1}{8}$$

6 (c) (iii)

HHH, HHT, HTH, THH, **HTT, THT, TTH, TTT**

No more than one head means one head or no heads are present.

$$p(\text{No more than one head}) = \frac{4}{8} = \frac{1}{2}$$

6 (c) (iv)

At least one head means you can have 1 head, 2 heads or 3 heads.

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$$p(\text{At least one head}) = \frac{7}{8}$$