COUNTING & PROBABILITY (Q 6, PAPER 2)

2005

6 (a) (i) Evaluate 6!

(ii) Evaluate $\binom{12}{3}$.

(b) Ten teams take part in a competition. The teams are divided into two groups. Teams A, B, C, D and E are in group 1 and teams U, V, X, Y and Z are in group 2.
In the final, the winning team from group 1 plays the winning team from group 2.
Each team is equally likely to win its group.

(i) How many different team pairings are possible for the final?

(ii) What is the probability that team C plays team X in the final?

(iii) What is the probability that team A plays in the final?

(iv) What is the probability that team B does not play in the final?

(c) Seven horses run in a race.
All horses finish the race and no two horses finish the race at the same time.

(i) In how many different orders can the seven horses finish the race?

(ii) A person is asked to predict the correct order of the first three horses to finish the race. How many different such predictions can be made?

(iii) A person is asked to predict, in any order, the first three horses to finish the race. How many different such predictions can be made?

(iv) A person selects two of the seven horses at random. What is the probability that the selected horses are the first two horses to finish the race?

SOLUTION

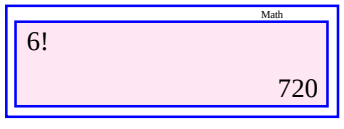
6 (a) (i)

6! means start with 6 and then multiply by 5 and then 4 all the way down to 1.

$6! = 6\times5\times4\times3\times2\times1 = 720$

CALCULATOR: Calculate 6!

6 SHIFT $x!$ =

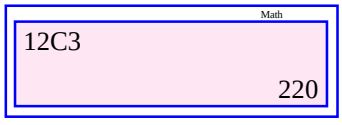


6 (a) (ii)

$$\binom{12}{3} = \frac{12\times11\times10}{3\times2\times1} = 220$$

CALCULATOR: Calculate $\binom{12}{3}$.

12 SHIFT nCr 3 =



6 (b) (i)

Group 1
A
B
C
D
E

Group 2
U
V
X
Y
Z

Make a list:
{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}
There are 25 possible pairings.

6 (b) (ii)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{C plays X}) = \frac{1}{25}$

6 (b) (iii)

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{A plays in final}) = \frac{5}{25} = \frac{1}{5}$

6 (b) (iv)

{(A, U), (A, V), (A, X), (A, Y), (A, Z), (B, U), (B, V), (B, X), (B, Y), (B, Z), (C, U), (C, V), (C, X), (C, Y), (C, Z), (D, U), (D, V), (D, X), (D, Y), (D, Z), (E, U), (E, V), (E, X), (E, Y), (E, Z)}

$p(\text{B does not play in final}) = \frac{20}{25} = \frac{4}{5}$

6 (c) (i)

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of ways in which you can arrange 7 different objects all taken, no repeats = 7!
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

6 (c) (ii)

The number of arrangements of n different objects taking r at a time with no repeats $= {}^nP_r$ 2

You are asked the number of ways in which you can pick 3 horses from 7 horses where order is important.

${}^7P_3 = 7 \times 6 \times 5 = 210$

6 (c) (iii)

The number of selections of n different objects taking r at a time $= {}^{n}C_{r} = \binom{n}{r}$ 1

You are asked the number of ways in which you can select 3 horses from 7 horses where order is not important.

$${}^{7}C_{3} = \binom{7}{3} = \frac{7\times6\times5}{3\times2\times1} = 35$$

6 (c) (iv)

How many ways can 2 horses be picked selected from 7 horses where order is not important?

$${}^{7}C_{2} = \binom{7}{2} = \frac{7\times6}{2\times1} = 21$$

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

As there are 21 possibilities for selecting 2 horses out of 7 horses, there is a 21 to 1 chance of these 2 horses finishing in the first two places.

p(2 particular horses are the first two in the race) $= \frac{1}{21}$