COUNTING & PROBABILITY (Q 6, PAPER 2)

2004

6 (a) The letters of the word CUSTOMER are arranged at random.

(i) How many different arrangements are possible?

(ii) How many of these arrangements begin with the letter C?

(b) A committee of 3 people is selected from a group of 15 doctors and 12 dentists.
In how many different ways can the 3 people be selected

(i) if there are no restrictions

(ii) if the selection must contain exactly 2 doctors

(iii) if the selection must contain at least 1 doctor and at least 1 dentist

(iv) if the selection must contain one specific doctor and one specific dentist?

(c) Four cards, numbered 2, 3, 4, 5 respectively, are shuffled and then placed in a row with the numbers visible.
Find the probability that

(i) the numbers shown are in the order: 5, 4, 3, 2

(ii) the first and second numbers are both even

(iii) the sum of the two middle numbers is 7.

SOLUTION

6 (a) (i)

The number of arrangements of n different objects all taken, no repeats = n! 3

The number of arrangements of 8 different letters all taken, no repeats = 8!

$$8! = 8\times7\times6\times5\times4\times3\times2\times1 = 40{,}320$$

CALCULATOR: Calculate 8!

8 SHIFT $x!$ =

Math
8!
40,320

OR

There are 8 ways to fill the first box. Once this is filled, there are 7 ways to fill the second box and so on.

Number of ways = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320

6 (a) (ii)

There is only one way to fill the first box (with the letter C). Once this is filled, there are 7 ways to fill the second box and so on.

Number of ways = 1 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5,040

C							

6 (b) (i)

The number of selections of n different objects taking r at a time $= {}^{n}C_r = \binom{n}{r}$ **1**

The number of ways of selecting 3 people from 27 people is: ${}^{27}C_3 = \binom{27}{3} = \frac{27 \times 26 \times 25}{3 \times 2 \times 1} = 2{,}925$

15 Doctors 12 Dentists

↓

3 Places

Calculator: Calculate ${}^{27}C_3$.

27 SHIFT nCr 3 =

Math

27C3

2,925

6 (b) (ii)

You need to select 2 doctors from 15 doctors **AND** 1 dentist from 12 dentists.

$${}^{15}C_2 \times {}^{12}C_1 = \left(\frac{15 \times 14}{2 \times 1}\right) \times \left(\frac{12}{1}\right) = 1{,}260$$

15 Doctors 12 Dentists

↓ ↓

2 Places 1 Place

Note: **AND** means you multiply.

6 (b) (iii)

At least one doctor and at least one dentist can mean 2 doctors and 1 dentist **OR** 1 doctor and 2 dentists.

2 doctors and 1 dentist: ${}^{15}C_2 \times {}^{12}C_1 = 1{,}260$

1 dentist and 2 doctors: ${}^{15}C_1 \times {}^{12}C_2 = \left(\frac{15}{1}\right) \times \left(\frac{12 \times 11}{2 \times 1}\right) = 990$

15 Doctors 12 Dentists

↓ ↓

2 Places 1 Place

OR

15 Doctors 12 Dentists

↓ ↓

1 Place 2 Places

Note: **OR** means you add.

2 doctors and 1 dentist **OR** 1 dentist and 2 doctors = 1,260 + 990 = 2,250

6 (b) (iv)

If one specific doctor is chosen and one specific dentist is chosen you are left to pick one person from 14 doctors and 11 dentists (25 people).

$${}^{25}C_1 = \left(\frac{25}{1}\right) = 25$$

14 Doctors 11 Dentists

↓

1 Place

6 (c) (i)

You can do this question the long way by writing out all the possibilities or the shorter way by using some formulae.

2 3 4 5	3 2 4 5	4 3 2 5	5 3 4 2
2 3 5 4	3 2 5 4	4 3 5 2	5 3 2 4
2 4 5 3	3 4 5 2	4 2 5 3	5 4 2 3
2 4 3 5	3 4 2 5	4 2 3 5	5 4 3 2
2 5 4 3	3 5 4 2	4 5 2 3	5 2 4 3
2 5 3 4	3 5 2 4	4 5 3 2	5 2 3 4

Long way: There are 24 possibilities. The is one possibility with the order as 5, 4, 3, 2.

$$p(E)=\frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

$p(5,4,3,2)=\frac{1}{24}$

Short way:

The number of arrangements of n different objects taking r at a time with no repeats $= {}^{n}P_{r}$ 2

How many ways can you arrange 4 different objects, all taken, no repeats (order is important)?

${}^{4}P_{3}=4\times3\times2\times1=24$

5, 4, 3, 2 is one such arrangement.

$\therefore\ {}^{4}P_{3}=4\times3\times2\times1=24$

6 (c) (ii)

Long way:

2 3 4 5	3 2 4 5	4 3 2 5	5 3 4 2
2 3 5 4	3 2 5 4	4 3 5 2	5 3 2 4
2 4 5 3	3 4 5 2	4 2 5 3	5 4 2 3
2 4 3 5	3 4 2 5	4 2 3 5	5 4 3 2
2 5 4 3	3 5 4 2	4 5 2 3	5 2 4 3
2 5 3 4	3 5 2 4	4 5 3 2	5 2 3 4

As you can see there are 4 arrangements out of 24 arrangments where the first and second numbers are even.

$p(\text{First 2 numbers are even})=\frac{4}{24}=\frac{1}{6}$

Short way:
Use the multiplication principle.
There are two even numbers. The first box must be even so there are 2 ways to fill the first box.
The second box must also be even. There is only one way to fill the second box once the first box is filled.
There are two ways to fill the third box as there are only two numbers left once the first two are filled.
Finally there is one way to fill the last box.

Number of ways = $2 \times 1 \times 2 \times 1 = 4$

Must be even

6 (c) (iii)

The best way to do this is by listing all the possibilities.

2 3 4 5	3 2 4 5	4 3 2 5	5 3 4 2
2 3 5 4	3 2 5 4	4 3 5 2	5 3 2 4
2 4 5 3	3 4 5 2	4 2 5 3	5 4 2 3
2 4 3 5	3 4 2 5	4 2 3 5	5 4 3 2
2 5 4 3	3 5 4 2	4 5 2 3	5 2 4 3
2 5 3 4	3 5 2 4	4 5 3 2	5 2 3 4

As you can see there are eight possible arrangements where the two middle numbers add up to 7.

$p(\text{Sum of middle two numbers is 7}) = \frac{8}{24} = \frac{1}{3}$