## Counting \& Probability (Q 6, Paper 2)

2003
6 (a) I write down today's date as 09062003 and then select one of the digits at random.
(i) What is the probability that I select the 9 ?
(ii) What is the probability that I select an odd digit?
(b) Two women, Ann and Bríd, and two men, Con and Declan, sit in a row for a photograph.
(i) How many different arrangements of the four people are possible?
(ii) Write out the four possible arrangements that have the two women in the middle.
(iii) If the arrangement of the four people is chosen at random from all of the possible arrangements, what is the probability that the two women will be in the middle?
(c) In a certain school the examination subjects for senior students are grouped as follows:

| Compulsory <br> Subjects | Block A | Block B | Block C |
| :--- | :--- | :--- | :--- |
| Irish <br> English <br> mathematics | French <br> German | biology <br> home economics <br> construction studies <br> accounting | business organisation <br> history <br> physics |

As well as taking all three of the compulsory subjects, each student must choose one subject from Block A, two from Block B and one from Block C.
(i) In choosing two subjects from Block B, how many different selections are possible?
(ii) In choosing the full range of subjects, how many different selections are possible?
(iii) One student has already decided to do German and construction studies. How many different selections of the remaining subjects are possible for this student?
(iv) If the student referred to in part (iii) selects her remaining subjects at random, what is the probability that she will select both biology and physics?

## Solution

6 (a) (i)

$$
p(E)=\frac{\text { Number of desired outcomes }}{\text { Total possible number of outcomes }}
$$

There are 8 digits in total. There is one 9 .
$p($ Selecting a 9$)=\frac{\text { No. of 9's }}{\text { No. of digits }}=\frac{1}{8}$
6 (a) (ii)
$p($ Selecting an Odd Digit $)=\frac{\text { No. of Odd Digits }}{\text { No. of digits }}=\frac{2}{8}=\frac{1}{4}$
6 (b) (i)
You can do this question the long way by writing out all the possibilities or the shorter way by using some formulae.
Long way:

| A | B | C | D | B | A | C | D | C | B | A | D | D | B | C | A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | D | C | B | A | D | C | C | B | D | A | D | B | A | C |
| A | C | B | D | B | C | A | D | C | A | B | D | D | C | B | A |
| A | C | D | B | B | C | D | A | C | A | D | B | D | C | A | B |
| A | D | B | C | B | D | A | C | C | D | B | A | D | A | B | C |
| A | D | C | B | B | D | C | A | C | D | A | B | D | A | C | B |

You can see there are 24 arrangements of 4 people.
Short way:

$$
\begin{aligned}
& \text { The number of arrangements of } n \text { different } \\
& \text { objects taking } r \text { at a time with no repeats }={ }^{n} P_{r}
\end{aligned}
$$

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How many ways can you arrange 4 different objects, all taken, no repeats (order is important)?
${ }^{4} P_{3}=4 \times 3 \times 2 \times 1=24$
4 (b) (ii)
(Con, Ann, Brid, Declan), (Declan, Ann, Brid, Con),
(Con, Brid, Ann, Declan), (Declan, Brid, Ann, Con)
4 (b) (iii)

$$
\begin{equation*}
p(E)=\frac{\text { Number of desired outcomes }}{\text { Total possible number of outcomes }} \tag{4}
\end{equation*}
$$

$p(2$ women in the middle $)=\frac{4}{24}=\frac{1}{6}$

6 (c) (i)

| Compulsory <br> Subjects | Block A | Block B | Block C |
| :--- | :--- | :--- | :--- |
| Irish <br> English <br> mathematics | French <br> German | biology <br> home economics <br> construction studies <br> accounting | business organisation <br> history <br> physics |


| [Must take all 3] [Must choose |
| :---: |
| 1 from 2] | | [Must choose 2 |
| :---: |
| from 4] |

[Must choose 1 from 2]

How many ways (order is not important) can 2 subjects be selected from 4 subjects?
${ }^{4} C_{2}=\frac{4 \times 3}{2 \times 1}=6$

The number of selections of $n$ different objects taking $r$ at a time $={ }^{n} C_{r}=\binom{n}{r}$

Calculator: Calculate ${ }^{4} C_{2}$.

## 4 SHIFT $\mathrm{nCr} 2=$



6 (c) (ii)
Compulsory subjects: You need to select 3 from 3 (no choice) AND
Block A: You need to select 1 from 2 AND
Block B: You need to select 2 from 4 AND
Block C: You need to select 1 from 3.
Note: AND means multiply.
No. of different selections $={ }^{3} C_{3} \times{ }^{2} C_{1} \times{ }^{4} C_{2} \times{ }^{3} C_{1}=1 \times 2 \times 6 \times 3=36$
6 (c) (iii)

| Compulsory <br> Subjects | Block A | Block B | Block C |
| :--- | :--- | :--- | :--- |
| Irish <br> English <br> mathematics | French <br> German | biology <br> home economics <br> construction studies <br> accounting | business organisation <br> history <br> physics |

[Choice made] [Choice made] | [Must choose 1 |
| ---: |
| from 3] |

[Must choose 1 from 2]

No. of different selections $=1 \times 1 \times{ }^{3} C_{1} \times{ }^{3} C_{1}=1 \times 1 \times 3 \times 3=9$

## 6 (c) (iv)

There are 9 possible selections. Selecting biology and physics is one such selection.
$p($ Physics and Biology $)=\frac{1}{9}$

