COUNTING & PROBABILITY (Q 6, PAPER 2)

2002

6 (a) There are eight questions on an examination paper.

(i) In how many different ways can a candidate select six questions?

(ii) In how many different ways can a candidate select six questions if one particular question must always be selected?

(b) A meeting is attended by 23 men and 21 women.
Of the men, 14 are married and the others are single.
Of the women, 8 are married and the others are single.

(i) A person is picked at random. What is the probability that the person is a woman?

(ii) A person is picked at random. What is the probability that the person is married?

(iii) A man is picked at random. What is the probability that he is married?

(iv) A woman is picked at random. What is the probability that she is single?

(c) The digits 0, 1, 2, 3, 4, 5 are used to form four-digit codes. A code cannot begin with 0 and no digit is repeated in any code.

(i) Write down the largest possible four-digit code.

(ii) Write down the smallest possible four-digit code.

(iii) How many four-digit codes can be formed?

(iv) How many of the four-digit codes are greater than 4000?

SOLUTION

6 (a) (i)

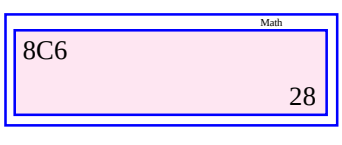
The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

In how many ways can you select 6 questions out of 8 questions (order is not important)?

$${}^8C_6 = \binom{8}{6} = \frac{8\times7\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times2\times1} = \frac{8\times7}{2\times1} = 28 \quad [\text{NOTE: } \binom{8}{6} = \binom{8}{2}.]$$

CALCULATOR: Calculate ${}^8C_6$.

8 SHIFT nCr 6 =



6 (a) (ii)

If one question must be answered, this means you have to select 5 questions from 7 questions.

$${}^7C_5 = \binom{7}{5} = \binom{7}{2} = \frac{7\times6}{2\times1} = 21$$

6 (b)

Draw up a table containing all the information.

	Married	Single
Men (23)	14	7
Women (21)	8	13

Total number of people: 44
Total number of married people: 22
Total number of single people: 20

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \; 4$$

6 (b) (i)

$$p(\text{Woman}) = \frac{\text{No. of women}}{\text{No. of people}} = \frac{21}{44}$$

6 (b) (ii)

$$p(\text{Married person}) = \frac{\text{No. of married people}}{\text{No. of people}} = \frac{22}{44} = \frac{1}{2}$$

6 (b) (iii)

$$p(\text{Married man}) = \frac{\text{No. of married men}}{\text{No. of men}} = \frac{14}{23}$$

6 (b) (iv)

$$p(\text{Single woman}) = \frac{\text{No. of single women}}{\text{No. of women}} = \frac{13}{21}$$

6 (c) (i)

5 digits: 0, 1, 2, 3, 4, 5
0 cannot be in the first position.
No repeats.
Largest possible number: **5** **4** **3** **2**

6 (c) (ii)

Smallest possible number: **1** **0** **2** **3**

6 (c) (iii)

Number of 4 digit codes:

Number of ways = 5 × 5 × 4 × 3 = 300

Cannot be a zero — □ □ □ □

Can be a zero but not what is in the first box

6 (c) (iv)

Number of 4 digit codes greater than 4000:

The first box must be filled with a 4 or 5 but not a zero (2 ways).

The second box can be filled 5 ways, the third 4 ways and so on.

Number of ways = $2 \times 5 \times 4 \times 3 = 120$

Must be a 4 or 5 ——