Counting & Probability (Q 6, Paper 2)

2001

6 (a) Sarah and Jim celebrate their birthdays in a particular week (Monday to Sunday inclusive).
Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that
(i) Sarah's birthday is on Friday

(ii) Sarah's birthday and Jim's birthday are both on Friday?

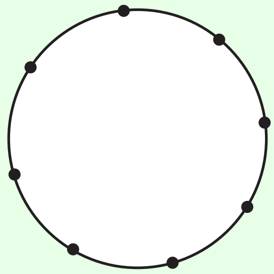
(b) (i) How many different arrangements can be made using all the letters of the word IRELAND?

(ii) How many arrangements begin with the letter I?

(iii) How many arrangements end with the word LAND?

(iv) How many begin with I and end with LAND?

(c) (i) Eight points lie on a circle, as in the diagram.
How many different lines can be drawn by joining any two of the eight points?



(ii) Find the value of the natural number n such that

$$\binom{n}{2} = 105.$$

[Note: $\binom{n}{2}$ may also be written as ${}^nC_2$.]

Solution

6 (a) (i)

There are 7 days in the week.

p(Sarah's birthday is on Friday) $= \frac{1}{7}$

6 (a) (ii)

$p(A \text{ and then } B) = p(A) \times p(B)$ **5**

p(Sarah's birthday is on Friday) $= \frac{1}{7}$

p(Jim's birthday is on Friday) $= \frac{1}{7}$

p(Sarah's and Jim's birthday is on Friday) $= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

6 (b) (i)

There are 7 different letters in the word IRELAND.

MULTIPLICATION PRINCIPLE:

There are 7 ways to fill the first box. Once this box is filled, there are 6 ways to fill the second box and so on.

Number of ways = $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}040$

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 7 different letters all taken, no repeats = 7!

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$

CALCULATOR: Calculate 7!

7 SHIFT $x!$ =

Math

7!

5,040

6 (b) (ii)

Number of ways = $1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

I						

There is only one way to fill the first box (with an **I**).

Once this is filled, there are 6 ways to fill the second box and so on.

6 (b) (iii)

Number of ways = $3 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 = 6$

			L	**A**	**N**	**D**

There is only one way to fill the last four boxes.

Once these are filled, there are 3 ways to fill the first box, 2 ways to fill the second box and one way to fill the third box.

6 (b) (iv)

Number of ways = $1 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 = 2$

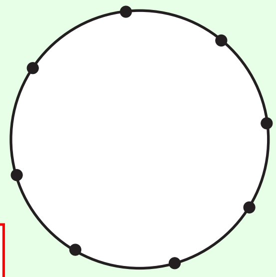
I			**L**	**A**	**N**	**D**

There is one way to fill the first box and the last four boxes. This means there are 2 letters left to fill the second box. Once this is filled, there is only one letter left to fill the third box.

6 (c) (i)

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ **1**

Lines are formed by selecting points in pairs. There are 8 points. In how many ways can you select 2 points from 8 different points?



$${}^8C_2 = \frac{8\times 7}{2\times 1} = 28$$

Calculator: Calculate ${}^8C_2$.

8 SHIFT **nCr** **2** **=**

Math

8C2

28

6 (c) (ii)

$$\binom{n}{2} = {}^nC_2 = 105$$

$$\Rightarrow \frac{n(n-1)}{2\times 1} = 105 \text{ [Multiply across by 2.]}$$

$$\Rightarrow n^2 - n = 210$$

$$\Rightarrow n^2 - n - 210 = 0$$

$$\Rightarrow (n-15)(n+14) = 0$$

$\therefore n = 15$ [Ignore the negative solution as n is a natural number (a whole positive number).]