

COUNTING & PROBABILITY (Q 6, PAPER 2)

2000

- 6 (a) To go to work, a woman can walk or travel by bus or travel by car with a neighbour. To return home, she can walk or travel by bus.
- (i) In how many different ways can the woman go to and return from work on any one day?
- (ii) List all of these different ways.
- (b) In a class, there are 15 boys and 13 girls. Four boys wear glasses and three girls wear glasses.
A pupil is picked at random from the class.
- (i) What is the probability that the pupil is a boy?
- (ii) What is the probability that the pupil wears glasses?
- (iii) What is the probability that the pupil is a boy who wears glasses?
- A girl is picked at random from the class.
- (iv) What is the probability that she wears glasses?
- (c) (i) How many different five-digit numbers can be formed from the digits 2, 3, 4, 5, 6? Each digit can be used once only in each number.
- (ii) How many of the numbers are even?
- (iii) How many of the numbers are less than 40 000?
- (iv) How many of the numbers are both even and less than 40 000?

SOLUTION

6 (a) (i)

There are 3 ways a woman can go to work **AND** 2 ways she can return home.

Number of ways the woman can go to and return from work = $3 \times 2 = 6$

NOTE: **AND** means multiply.

6 (a) (ii)

{(Walk, Walk), (Walk, Bus), (Bus, Walk), (Bus, Bus), (Car, Walk), (Car, Bus)}

6 (b)

Draw up a table containing all the information.

	Wear Glasses	Does not wear glasses
Boys (15)	4	11
Girls (13)	3	10

Total number of pupils: 28

Total number of pupils who wear glasses: 7

Total number of pupils who do not wear glasses: 21

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \dots\dots \textcircled{4}$$

6 (b) (i)

$$p(\text{Boy}) = \frac{\text{No. of boys}}{\text{No. of pupils}} = \frac{15}{28}$$

6 (b) (ii)

$$p(\text{Pupil who wears glasses}) = \frac{\text{No. of pupils who wear glasses}}{\text{No. of pupils}} = \frac{7}{28} = \frac{1}{4}$$

6 (b) (iii)

$$p(\text{Boy who wears glasses}) = \frac{\text{No. of boys who wear glasses}}{\text{No. of pupils}} = \frac{4}{28} = \frac{1}{7}$$

6 (b) (iv)

$$p(\text{Girl who wears glasses}) = \frac{\text{No. of girls who wear glasses}}{\text{No. of girls}} = \frac{3}{13}$$

6 (c) (i)

Multiplication Principle:

There are 5 digits in total. Therefore, there are 5 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

$$\text{Number of ways} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

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OR

$$\text{The number of arrangements of } n \text{ different objects all taken, no repeats} = n! \dots\dots \textcircled{3}$$

The number of arrangements of 5 different numbers all taken, no repeats = 5!

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

CALCULATOR: Calculate 5!

5 SHIFT x! =

5! 120

6 (c) (ii)

Always work on the box with the restriction first. The last box must be filled with an even digit. There are 3 ways to fill this box (with a 2, 4 or 6).

Once this is filled, there are 4 ways to fill the first box and so on.

Number of ways = $4 \times 3 \times 2 \times 1 \times 3 = 72$

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Must be an even digit

6 (c) (iii)

Always work on the box with the restriction first. The first box must be filled with a 2 or 3 (two ways).

Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = $2 \times 4 \times 3 \times 2 \times 1 = 48$

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Must be a 2 or 3

6 (c) (iv)

This can be a little tricky because the 2 is needed to make the number even and also to the make the number less than 40,000. Therefore, consider both possibilities for filling the first box.

Fill the first box with a 2. There is one way to fill it.

The last box must contain an even digit. There are two even digits left (a 4 or 6). Therefore, there are two ways to fill it.

Once these two boxes are filled, there are three ways to fill the second box and so on.

Number of ways = $1 \times 3 \times 2 \times 1 \times 2 = 12$

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Must be even but not a 2

OR

Fill the first box with a 3. There is one way to fill it.

The last box must contain an even digit. There are three even digits (a 2, 4 or 6). Therefore, there are three ways to fill it.

Once these two boxes are filled, there are three ways to fill the second box and so on.

Number of ways = $1 \times 3 \times 2 \times 1 \times 3 = 18$

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Must be even

OR means add. Add the two possibilities together.

Number of numbers less than 40,000 and even = $12 + 18 = 30$