

COUNTING & PROBABILITY (Q 6, PAPER 2)

1999

- 6 (a) (i) In how many ways can a team of 5 players be chosen from a panel of 8 players?
- (ii) If a certain player must be on the team, in how many ways can the team be then chosen.
- (b) (i) In how many different ways can the 5 letters of the word ANGLE be arranged?
- (ii) How many of these arrangements begin with a vowel?
- (iii) In how many of the arrangements do the two vowels come together?
- (c) Twelve blood samples are tested in a laboratory. Of these it is found that five blood samples are of type A, four of type B and the remaining three are of type O. Two blood samples are selected at random from the twelve. What is the probability that
- (i) the two samples are of type A
- (ii) one sample is of type B and the other sample is of type O
- (iii) the two sample are of the same blood type?

SOLUTION

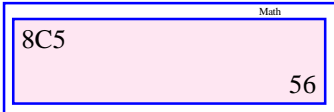
6 (a) (i)

The number of selections of n different objects taking r at a time = ${}^n C_r = \binom{n}{r}$ 1

The number of selections of 8 different players taking 5 at a time = ${}^8 C_5 = \binom{8}{5}$.

$${}^8 C_5 = \binom{8}{5} = \frac{8 \times 7 \times 6 \times \cancel{5} \times \cancel{4}}{\cancel{5} \times \cancel{4} \times 3 \times 2 \times 1} = 56$$

CALCULATOR: Calculate ${}^8 C_5$.



The calculator interface shows the input '8C5' and the result '56'. The 'Math' button is visible above the display.

6 (a) (ii)

If a certain player must be on the team, you need to choose 4 players from the remaining 7 players.

$${}^7 C_4 = \binom{7}{4} = \frac{7 \times 6 \times 5 \times \cancel{4}}{\cancel{4} \times 3 \times 2 \times 1} = 35$$

6 (b) (i)

Multiplication Principle:

There are 5 letters in total. Therefore, there are 5 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

$$\text{Number of ways} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

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OR

The number of arrangements of n different objects all taken, no repeats = $n!$ **3**

The number of arrangements of 5 different letters all taken, no repeats = $5!$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

CALCULATOR: Calculate $5!$



The image shows a calculator interface with buttons for '5', 'SHIFT', 'x!', and '='. The display shows '5!' and the result '120'.

6 (b) (ii)

There are 2 vowels (A and E). There are 2 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

$$\text{Number of ways} = 2 \times 4 \times 3 \times 2 \times 1 = 48$$

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Must be a vowel

6 (b) (iii)

Glue the two vowels together and treat as a single unit.

One such arrangement **AE** **N** **G** **L**

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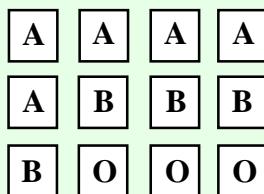
Must be a vowel

There are $4!$ ways of arranging 4 objects AND then there are $2!$ ways of arranging the two objects glued together.

$$\text{No. of arrangements of the 5 letters with the vowels side by side} = 4! \times 2! = 24 \times 2 = 48$$

NOTE: The word AND means multiply.

6 (c) (i)



$$p(\mathbf{A} \text{ and } \mathbf{A}) = p(\mathbf{A}) \times p(\mathbf{A})$$

$$p(A \text{ and then } B) = p(A) \times p(B) \dots\dots \mathbf{5}$$

$$\text{First pick: } p(\mathbf{A}) = \frac{\text{No. of } \mathbf{A}'\text{s}}{\text{No. of samples}} = \frac{5}{12}$$

For the second pick, there are 4 **A**'s left out of 11 samples.

$$\text{Second pick: } p(\mathbf{A}) = \frac{\text{No. of } \mathbf{A}'\text{s}}{\text{No. of samples}} = \frac{4}{11}$$

$$p(\mathbf{A} \text{ and } \mathbf{A}) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$$

6 (c) (ii)

$$p(\mathbf{B} \text{ and } \mathbf{O}) = p(\mathbf{B}) \times p(\mathbf{O}) \times 2$$

You need to multiply your answer by two because you could pick **B** first and **O** second or **O** first and **B** second.

$$\text{First pick: } p(\mathbf{B}) = \frac{\text{No. of } \mathbf{B}'\text{s}}{\text{No. of samples}} = \frac{4}{12} = \frac{1}{3}$$

For the second pick there are eleven samples left to pick from.

$$\text{Second pick: } p(\mathbf{O}) = \frac{\text{No. of } \mathbf{O}'\text{s}}{\text{No. of samples}} = \frac{3}{11}$$

$$p(\mathbf{B} \text{ and } \mathbf{O}) = \frac{1}{3} \times \frac{3}{11} \times 2 = \frac{2}{11}$$

6 (c) (iii)

$$p(\text{Same sample}) = p(2 \mathbf{A}'\text{s}) \text{ OR } p(2 \mathbf{B}'\text{s}) \text{ OR } p(2 \mathbf{O}'\text{s})$$

[OR means add the probabilities together.]

$$p(\text{Same sample}) = p(2 \mathbf{A}'\text{s}) + p(2 \mathbf{B}'\text{s}) + p(2 \mathbf{O}'\text{s})$$

$$p(\mathbf{A} \text{ and } \mathbf{A}) = \frac{5}{33}$$

$$p(\mathbf{B} \text{ and } \mathbf{B}) = \frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$$

$$p(\mathbf{O} \text{ and } \mathbf{O}) = \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$$

$$p(\text{Same sample}) = \frac{5}{33} + \frac{1}{11} + \frac{1}{22} = \frac{19}{66}$$