COUNTING & PROBABILITY (Q 6, PAPER 2)

1999

6 (a) (i) In how many ways can a team of 5 players be chosen from a panel of 8 players?

(ii) If a certain player must be on the team, in how many ways can the team be then chosen.

(b) (i) In how many different ways can the 5 letters of the word ANGLE be arranged?

(ii) How many of these arrangements begin with a vowel?

(iii) In how many of the arrangements do the two vowels come together?

(c) Twelve blood samples are tested in a laboratory. Of these it is found that five blood samples are of type A, four of type B and the remaining three are of type O.
Two blood samples are selected at random from the twelve.
What is the probability that

(i) the two samples are of type A

(ii) one sample is of type B and the other sample is of type O

(iii) the two sample are of the same blood type?

SOLUTION

6 (a) (i)

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ **1**

The number of selections of 8 different players taking 5 at a time $= {}^8C_5 = \binom{8}{5}$.

$${}^8C_5 = \binom{8}{5} = \frac{8\times7\times6\times\cancel{5}\times\cancel{4}}{\cancel{5}\times\cancel{4}\times3\times2\times1} = 56$$

CALCULATOR: Calculate ${}^8C_5$.

8 SHIFT nCr 5 =

Math
8C5
56

6 (a) (ii)

If a certain player must be on the team, you need to choose 4 players from the remaining 7 players.

$${}^7C_4 = \binom{7}{4} = \frac{7\times6\times5\times\cancel{4}}{\cancel{4}\times3\times2\times1} = 35$$

6 (b) (i)

Multiplication Principle:
There are 5 letters in total. Therefore, there are 5 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = $5 \times 4 \times 3 \times 2 \times 1 = 120$

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 5 different letters all taken, no repeats = 5!

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

CALCULATOR: Calculate 5!

5 SHIFT $x!$ =

Math
5!
120

6 (b) (ii)

There are 2 vowels (A and E). There are 2 ways to fill the first box. Once this is filled, there are 4 ways to fill the second box and so on.

Number of ways = $2 \times 4 \times 3 \times 2 \times 1 = 48$

Must be a vowel

6 (b) (iii)

Glue the two vowels together and treat as a single unit.

One such arrangement **AE** **N** **G** **L**

Must be a vowel

There are 4! ways of arranging 4 objects AND then there are 2! ways of arranging the two objects glued together.

No. of arrangements of the 5 letters with the vowels side by side $= 4! \times 2! = 24 \times 2 = 48$

NOTE: The word AND means multiply.

6 (c) (i)

A	**A**	**A**	**A**
A	**B**	**B**	**B**
B	**O**	**O**	**O**

$p(\textbf{A} \text{ and } \textbf{A}) = p(\textbf{A}) \times p(\textbf{A})$

$p(A \text{ and then } B) = p(A) \times p(B)$ 5

First pick: $p(\textbf{A}) = \dfrac{\text{No. of } \textbf{A}\text{'s}}{\text{No. of samples}} = \dfrac{5}{12}$

For the second pick, there are 4 **A**'s left out of 11 samples.

Second pick: $p(\textbf{A}) = \dfrac{\text{No. of } \textbf{A}\text{'s}}{\text{No. of samples}} = \dfrac{4}{11}$

$p(\textbf{A} \text{ and } \textbf{A}) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

6 (c) (ii)

$p(\textbf{B} \text{ and } \textbf{O}) = p(\textbf{B}) \times p(\textbf{O}) \times 2$

You need to multiply your answer by two because you could pick **B** first and **O** second or **O** first and **B** second.

First pick: $p(\textbf{B}) = \dfrac{\text{No. of } \textbf{B}\text{'s}}{\text{No. of samples}} = \dfrac{4}{12} = \dfrac{1}{3}$

For the second pick there are eleven samples left to pick from.

Second pick: $p(\textbf{O}) = \dfrac{\text{No. of } \textbf{O}\text{'s}}{\text{No. of samples}} = \dfrac{3}{11}$

$p(\textbf{B} \text{ and } \textbf{O}) = \frac{1}{3} \times \frac{3}{11} \times 2 = \frac{2}{11}$

6 (c) (iii)

p(Same sample) = p(2 **A**'s) OR p(2 **B**'s) OR p(2 **O**'s)
[OR means add the probabilities together.]

p(Same sample) = p(2 **A**'s) + p(2 **B**'s) + p(2 **O**'s)

$p(\textbf{A} \text{ and } \textbf{A}) = \frac{5}{33}$

$p(\textbf{B} \text{ and } \textbf{B}) = \frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$

$p(\textbf{O} \text{ and } \textbf{O}) = \frac{3}{12} \times \frac{2}{11} = \frac{1}{22}$

$p(\text{Same sample}) = \frac{5}{33} + \frac{1}{11} + \frac{1}{22} = \frac{19}{66}$