COUNTING & PROBABILITY (Q 6, PAPER 2)

1998

6 (a) One letter is chosen at random from the letters of the word LEAVING.
    (i) Find the probability that the letter chosen is L.

    (ii) Find the probability that the letter chosen is a vowel.

  (b) A committee of 4 people is to be formed from a group of 7 men and 6 women.
    (i) How many different committees can be formed?

    (ii) On how many of these committees is there an equal number of men and of women?

  (c) (i) How many different numbers, each with 3 digits or less, can be formed from the digits 1, 2, 3, 4, 5? Each digit can be used only once in each number.

    (ii) How many of the above numbers are even?

SOLUTION

6 (a) (i)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \cdots\cdots \quad \textbf{4}$$

$$p(\textbf{L}) = \frac{\text{No. of } \textbf{L's}}{\text{No. of letters}} = \frac{1}{7}$$

6 (a) (ii)

$$p(\text{Vowel}) = \frac{\text{No. of vowels}}{\text{No. of letters}} = \frac{3}{7}$$

6 (b) (i)

The number of selections of n different objects taking r at a time $= {}^nC_r = \begin{pmatrix} n \\ r \end{pmatrix}$ $\cdots\cdots$ **1**

In total there are 13 people (7 men and 6 women).
In how many ways can you pick 4 people on the committee from 13 people?

7 Men   6 Women
↓
4 Places
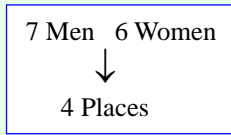
$${}^{13}C_4 = \begin{pmatrix} 13 \\ 4 \end{pmatrix} = \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = 715$$

CALCULATOR: Calculate ${}^{13}C_4$.

Math

13C4

714

13   SHIFT   nCr   **4**   =

6 (b) (ii)

You need to pick 2 men and 2 women for the committee to have an equal number of men and women.
How many ways can you pick 2 men from 7 men **AND** 2 women from 6 women?
Note: **AND** means multiply.

7 Men	6 Women
↓	↓
2 Men	2 Women

$$^7C_2 \times {}^6C_2 = \binom{7}{2} \times \binom{6}{2} = \left(\frac{7 \times 6}{2 \times 1}\right) \times \left(\frac{6 \times 5}{2 \times 1}\right) = 21 \times 15 = 315$$

6 (c) (i)

There are five digits (1, 2, 3, 4 and 5). How many three digit numbers, two digits numbers and single digit numbers can be formed from these five digits with no repeats?

THREE DIGIT NUMBERS:

There are 5 ways to fill the first box. Once this is filled there are 4 ways to fill the second box and three ways to fill the third box. Therefore, there are 60 possible three digit numbers.

Number of ways = 5 × 4 × 3 = 60

TWO DIGIT NUMBERS:

There are 5 ways to fill the first box. Once this is filled there are 4 ways to fill the second box. Therefore, there are 20 possible two digit numbers.

Number of ways = 5 × 4 = 20

ONE DIGIT NUMBERS:

Obviously, there are 5 one digit numbers.

The total number of numbers of 3 digits or less: 60 + 20 + 5 = 85

6 (c) (ii)

EVEN THREE DIGIT NUMBERS:

The last box must be filled with an even digit in order for the number to be even. There are two ways to fill the last box (with a 2 or 4). Once the last box is filled there are fours ways to fill the first box and three ways to fill the second box.

Number of ways = 4 × 3 × 2 = 24

Must be even

EVEN TWO DIGIT NUMBERS:

The last box must be filled with an even digit in order for the number to be even. There are two ways to fill the last box (with a 2 or 4). Once the last box is filled there are fours ways to fill the first box.

Number of ways = 4 × 2 = 8

Must be even

EVEN ONE DIGIT NUMBERS:

Obviously there are 2 even one digit numbers.

The total number of even numbers of 3 digits or less: 24 + 8 + 2 = 34