COUNTING & PROBABILITY (Q 6, PAPER 2)

1997

6 (a) A class of 29 students wins a prize. Two members of the class are chosen to receive the prize. How many different pairs of students can be chosen?

(b) (i) In how many different ways can the letters of the word CARPET be arranged?

(ii) How many of these arrangements begin with A?

(iii) In how many of the arrangements do the two vowels come together?

(c) Two people are chosen at random from a large crowd. Each person names the day of the week on which he or she was born. Assuming that each day is equally likely, what is the probability that
(i) both people were born on a Friday

(ii) one person was born on a Tuesday and the other was born on a Thursday

(iii) the two people were born on different days?

SOLUTION

6 (a)

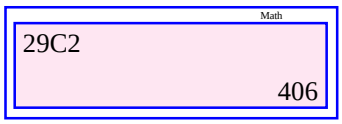
The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

In how many ways can 2 students be selected from 29 students (order is not important)?

$${}^{29}C_2 = \binom{29}{2} = \frac{29\times 28}{2\times 1} = 406$$

CALCULATOR: Calculate ${}^{29}C_2$.

29 SHIFT nCr 2 =



6 (b) (i)

USE THE MULTIPLICATION PRINCIPLE: There are 6 ways to fill the first box. Once this is filled, there are 5 ways to fill the second box and so on.

Number of ways = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

OR

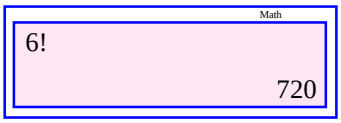
The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 6 different letters all taken, no repeats = 6!

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

CALCULATOR: Calculate 6!

6 SHIFT $x!$ =



6 (b) (ii)

Number of ways = $1 \times 5 \times 4 \times 3 \times 2 \times 1 = 120$

There is only one way to fill the first box (with the letter **A**). Once this box is filled there are only 5 ways to fill the second box. Once this box is filled there are only 4 ways to fill the third box and so on.

6 (b) (iii)

Glue the two vowels together and treat as a single unit.

One such arrangement 

There are 5! ways of arranging 5 objects **AND** then there are 2! ways of arranging the two objects glued together.

No. of arrangements of the 5 letters with the vowels side by side $= 5! \times 2! = 120 \times 2 = 240$

NOTE: The word **AND** means multiply.

6 (c) (i)

There are 7 days in the week. Call the two people A and B.

$$p(A \text{ and then } B) = p(A) \times p(B) \quad \ldots\ldots \quad 5$$

p(A has her birthday on Friday) $= \frac{1}{7}$

p(B has her birthday on Friday) $= \frac{1}{7}$

p(Birthdays are both on Friday) $= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

6 (c) (ii)

The probability that A's birthday is on a Tuesday is $\frac{1}{7}$. The probability that B's birthday is on a Thursday is $\frac{1}{7}$. However, it could be the other way round so multiply your answer by two.

p(Birthdays on Tuesday and Thursday) $= \frac{1}{7} \times \frac{1}{7} \times 2 = \frac{2}{49}$

6 (c) (iii)

A has her birthday on any day. The probability of A having her birthday on any day is $\frac{7}{7} = 1$.

The probability that B has her birthday on a different day to A is $\frac{6}{7}$.

p(Birthdays on different days) $= 1 \times \frac{6}{7} = \frac{6}{7}$