COUNTING & PROBABILITY (Q 6, PAPER 2)

1996

6 (a) A bag contains 24 beads of which 12 are red, 8 are blue and 4 are white.
A bead is taken at random from the bag.
What is the probability that the colour of the bead is
(i) blue
(ii) red or white?

(b) There are 5 horses, *A*, *B*, *C*, *D* and *E*, in a race. Each horse takes a different time to complete the race. On completing the race,
(i) in how many different placing arrangements can the 5 horses finish?

(ii) if *A* is placed first and *B* last, in how many different placing arrangements can the other horses finish?

(c) A committee of two people is chosen at random from 4 men and 5 women.
What is the probability that there will be one woman or two women on the committee?

SOLUTION

5 (a) (i)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} \quad \ldots\ldots \ 4$$

$$p(\text{Blue}) = \frac{\text{No. of blue discs}}{\text{No. of discs}} = \frac{8}{24} = \frac{1}{3}$$

5 (a) (ii)

$$p(\text{Red or White}) = \frac{\text{No. of Red and White discs}}{\text{No. of discs}} = \frac{16}{24} = \frac{2}{3}$$

5 (b) (i)

MULTIPLICATION PRINCIPLE: There are 5 horses that can finish in first place. Once a horse finishes in first place there are 4 horses that can finish in second place and so on.

Number of ways = 5 × 4 × 3 × 2 × 1 = 120

1st	2nd	3rd	4th	5th
☐	☐	☐	☐	☐

OR

The number of arrangements of n different objects all taken, no repeats = $n!$ 3

The number of arrangements of 5 different horses all taken, no repeats = 5!

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

CALCULATOR: Calculate 5!

5 SHIFT $x!$ =

Math
5!
120

5 (b) (ii)

Number of ways = 1 × 3 × 2 × 1 × 1 = 6

A				**B**
1st	2nd	3rd	4th	5th

There is one way to fill the first box (with **A**) and one way to fill the last box (with **B**). Once these are filled there are 3 horses left to fill the second box, 2 left to fill the third box and 1 horse left to fill the fourth box.

5 (c)

First, work out the total number of 2 person committees that can be picked from 9 people.

The number of selections of n different objects taking r at a time $= {}^nC_r = \binom{n}{r}$ 1

4 Men 5 Women
↓
2 Places

The number of selections of 9 different people taking 2 at a time:

$${}^9C_2 = \binom{9}{2} = \frac{9\times 8}{2\times 1} = 36$$

CALCULATOR: Calculate ${}^9C_2$.

9 SHIFT nCr 2 =

Math
9C2
36

Therefore, there are 36 possible committees.

Now work out how many of these committees have 1 woman or 2 women.
Consider the 2 possibilities:

How many ways can you pick one man from 4 men and 1 woman from 5 women?

4 Men 5 Women
↓ ↓
1 Man 1 Woman

$${}^4C_1 \times {}^5C_1 = 4\times 5 = 20$$

OR

How many ways can you pick no men from 4 men and 2 women from 5 women?

4 Men 5 Women
↓ ↓
0 Man 2 Women

$${}^4C_0 \times {}^5C_2 = 1\times 10 = 10$$

OR

NOTE: *OR* means add.

Therefore, the number of committees with 1 woman or 2 women = 20 + 10 = 30

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}}$$ 4

$$p(\text{1 woman or 2 woman}) = \frac{\text{No. of committees with 1 or 2 women}}{\text{No. of committees}} = \frac{30}{36} = \frac{5}{6}$$