Linear Programming (Q 11, Paper 2)

Lesson No. 3: Practical Linear Programming Problems

2007

11 (b) A developer is planning a holiday complex of cottages and apartments.
Each cottage will accommodate 3 adults and 5 children and each apartment will accommodate 2 adults and 2 children.
The other facilities in the complex are designed for a maximum of 60 adults and a maximum of 80 children.

(i) Taking x as the number of cottages and y as the number of apartments, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental income per night will be €65 for a cottage and €40 for an apartment, how many of each should the developer include in the complex to maximise potential rental income?

(iii) If the construction costs are €200 000 for a cottage and €120 000 for an apartment, how many of each should the developer include in the complex to minimise construction costs?

Solution

11 (b)

Maximising and Minimising Problems

Steps
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of cottages
Let y = Number of apartments

2.

	Cottages	Apartments	Restriction
Adults	$3x$	$2y$	60
Children	$5x$	$2y$	80

Adults inequality: $3x+2y\le 60$

Children inequality: $5x+2y\le 80$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

Cont......

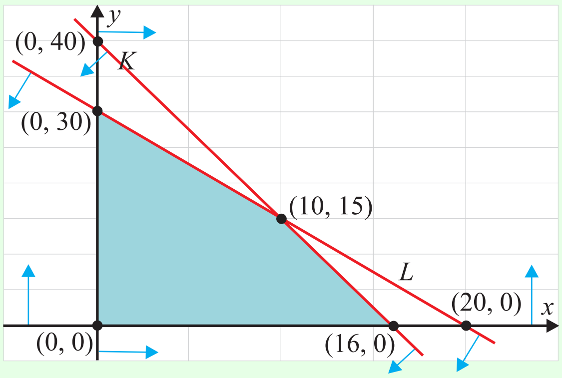
3. Plot the four inequalities.

Graph $3x+2y \le 60$. Draw the line $3x+2y=60$. Call it K.

Intercepts: (0, 30), (20, 0). Test with (0, 0) $\Rightarrow 3(0)+2(0)=0 \le 60$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x+2y \le 80$. Draw the line $5x+2y=80$. Call it L.

Intercepts: (0, 40), (16, 0). Test with (0, 0) $\Rightarrow 5(0)+2(0)=0 \le 80$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (16, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$3x+2y=60 \ldots \mathbf{(1)}$$
$$5x+2y=80 \ldots \mathbf{(2)} \quad (\times -1)$$

$$\begin{aligned} 3x+2y &= 60 \\ -5x-2y &= -80 \\ \hline -2x \quad &= -20 \Rightarrow x=10 \end{aligned}$$

Substitute x = 10 back into Eqn. (**1**).

$\Rightarrow 3(10)+2y=60 \Rightarrow 2y=30 \Rightarrow y=15$

Therefore (10, 15) is the final vertex of the region.

5. Rental income = 65x + 40y is the function to be minimised.

	65x + 40y	**Income**
(0, 0)	65(0) + 40(0)	€0
(0, 30)	65(0) + 40(30)	€1200
(10, 15)	65(10) + 40(15)	€**1250**
(16, 0)	65(16) + 40(0)	€1040

Therefore, 10 cottages and 15 apartments give the maximum rental income.

Construction costs = 200000x + 120000y is the function to be minimised.

	200000x + 120000y	**Cost**
(0, 0)	200000(0) + 120000(0)	€0
(0, 30)	200000(0) + 120000(30)	€3,600,000
(10, 15)	200000(10) + 120000(15)	€3,800,000
(16, 0)	200000(16) + 120000(0)	€**3,200,000**

Therefore, 16 cottages and 0 apartments give the minimum construction costs.

2006

11 (b) Due to a transport disruption, a bus company is contracted at short notice to carry up to 1500 passengers to complete their journey. Passengers not carried by this company will be carried by a taxi company.

The bus company has available standard buses and mini-buses. Each standard bus carries 60 passengers and each mini-bus carries 30 passengers.

Each bus is operated by one driver and there are at most 30 drivers available.

(i) Taking x as the number of standard buses and y as the number of mini-buses, write down two inequalities in x and y and illustrate them on graph paper.

(ii) The operating profit for the journey is €80 for a standard bus and €50 for a minibus. How many of each type of bus should be used in order to maximise the profit?

(iii) If the bus company paid each driver a bonus for working at short notice, the operating profit for each bus would be reduced by €30. By how much would this decrease the maximum profit available to the company?

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of standard buses
Let y = Number of mini-buses

2.

	Standard buses	Mini-buses	Restriction
Passengers	$60x$	$30y$	1500
Drivers	x	y	30

Passsengers inequality: $60x + 30y \le 1500 \Rightarrow 2x + y \le 50$

Drivers inequality: $x + y \le 30$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.

Graph $2x + y \le 50$. Draw the line $2x + y = 50$. Call it K.

Intercepts: (0, 50), (25, 0). Test with (0, 0) $\Rightarrow 2(0) + (0) = 0 \le 50$. This is true.

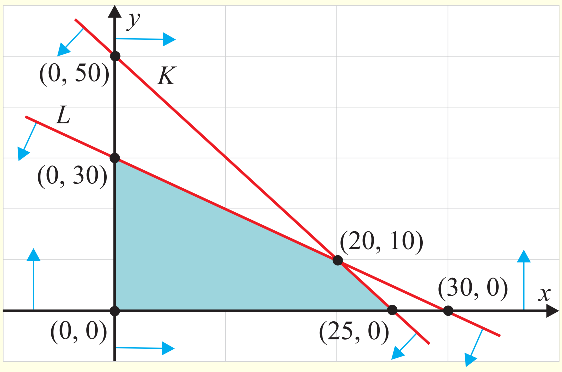
Shade the side of the line that contains (0, 0).

CONT......

Graph $x + y \leq 30$. Draw the line $x + y = 30$. Call it L.

Intercepts: (0, 30), (30, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0 \leq 30$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (25, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x + y = 50 \ldots(\mathbf{1})$$
$$x + y = 30 \ldots(\mathbf{2}) \quad (\times -1)$$

$$\begin{aligned} 2x + y &= 50 \\ -x - y &= -30 \\ \hline x \quad &= 20 \end{aligned}$$

Substitute x = 20 back into Eqn. (**2**).

$\Rightarrow (20) + y = 30 \Rightarrow y = 10$

Therefore (20, 10) is the final vertex of the region.

5. Profit = 80x + 50y is the function to be maximised.

	80x + 50y	**Profit**
(0, 0)	80(0) + 50(0)	€0
(0, 30)	80(0) + 50(30)	€1500
(20, 10)	80(20) + 50(10)	€**2100**
(25, 0)	80(25) + 50(0)	€2000

Therefore, 20 standard buses and 10 mini-buses give the maximum profit.

Answers

11 (b) (i) $x + y \leq 30,\ 2x + y \leq 50$

11 (b) (ii) $x = 20,\ y = 10$

11 (b) (iii)

Profit = 50x + 20y is the function to be maximised now that the operating profit for each bus is reduced by €30.

	50x + 20y	**Profit**
(0, 0)	50(0) + 20(0)	€0
(0, 30)	50(0) + 20(30)	€600
(20, 10)	50(20) + 20(10)	€1200
(25, 0)	50(25) + 20(0)	€**1250**

The maximum profit would now be for 25 standard buses and 0 mini-buses.
Therefore, the decrease in profit would €**2100** – €**1250** = €**850**

2005

11 (b) A manufacturer of garden furniture produces plastic chairs and tables. Each chair requires 2 kg of raw material and each table requires 5 kg of raw material. In any working period the raw material used cannot exceed 800 kg.

Each chair requires 4 minutes of machine time and each table requires 4 minutes of machine time. The total machine time available in any working period is 1000 minutes.

(i) Taking x as the number of chairs and y as the number of tables, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The manufacturer sells each chair for €20 and each table for €40. How many of each should be produced in each working period to maximise income?

(iii) The manufacturer's costs for each chair are €17 and for each table are €34.70. Express the profit as a percentage of income, assuming the income has been maximised.

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of chairs
Let y = Number of tables

2.

	Chairs	Tables	Restriction
Raw material	$2x$	$5y$	800
Time	$4x$	$4y$	1000

Raw material inequality: $2x+5y\le 800$

Time inequality: $4x+4y\le 1000 \Rightarrow x+y\le 250$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

CONT......

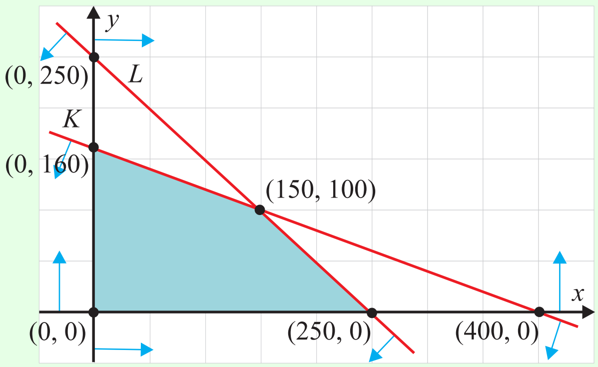
3. Plot the four inequalities.

Graph $2x+5y\leq 800$. Draw the line $2x + 5y = 800$. Call it K.

Intercepts: (0, 160), (400, 0). Test with (0, 0) $\Rightarrow 2(0)+5(0)=0\leq 800$. This is true.
Shade the side of the line that contains (0, 0).

Graph $x+y\leq 250$. Draw the line $x + y = 250$. Call it L.

Intercepts: (0, 250), (250, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0\leq 250$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 160) and (250, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x+5y=800\ldots\mathbf{(1)}$$
$$x+y=250\ldots\mathbf{(2)}\quad(\times -2)$$

$$2x+5y=800$$
$$-2x-2y=-500$$
$$3y=300\Rightarrow y=100$$

Substitute $y = 100$ back into Eqn. **(2)**.

$\Rightarrow x+(100)=250\Rightarrow x=250-100=150$

Therefore (150, 100) is the final vertex of the region.

5. Income = $20x + 40y$ is the function to be minimised.

	$20x + 40y$	**Income**
(0, 0)	20(0) + 40(0)	€0
(0, 160)	20(0) + 40(160)	€6400
(150, 100)	20(150) + 40(100)	€**7000**
(250, 0)	20(250) + 40(0)	€5000

Therefore, 150 chairs and 100 tables give the maximum income.

11 (b) (i) $2x+5y\leq 800,\ x+y\leq 250$

11 (b) (ii) $x = 150, y = 100$

11 (b) (iii)

Manufacturing costs $=150\times €17+100\times €34.70=€6020$

Profit $=€7000-€6020=€980$

Profit as a percentage of income $=\dfrac{€980}{€7000}\times 100\%=14\%$

2004

11 (b) A shop-owner displays videos and DVDs in his shop.
Each video requires 720 cm^3 of display space and each DVD requires 360 cm^3 of display space. The available display space cannot exceed 108 000 cm^3. The shopowner buys each video for €6 and each DVD for €8. He does not wish to spend more than €1200.

(i) Taking x as the number of videos and y as the number of DVDs, write down two inequalities in x and y and illustrate these on graph paper.

During a DVD promotion the selling price of a video is €11 and of a DVD is €10. Assuming that the shop-owner can sell all the videos and DVDs,

(ii) how many of each type should he display in order to maximise his income?

(iii) how many of each type should he display in order to maximise his profit?

SOLUTION

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of videos
Let y = Number of DVDs

2.

	Videos	DVDs	Restriction
Space	$720x$	$360y$	108,000
Cost	$6x$	$8y$	1200

Space inequality: $720x+360y \le 108000 \Rightarrow 2x+y \le 300$

Cost inequality: $6x+8y \le 1200 \Rightarrow 3x+4y \le 600$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.

Graph $2x+y \le 300$. Draw the line $2x + y = 300$. Call it K.

Intercepts: (0, 300), (150, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0 \le 300$. This is true.
Shade the side of the line that contains (0, 0).

Graph $3x+4y \le 600$. Draw the line $3x + 4y = 600$. Call it L.

Intercepts: (0, 150), (200, 0). Test with (0, 0) $\Rightarrow 3(0)+4(0)=0 \le 600$.
This is true. Shade the side of the line that contains (0, 0).

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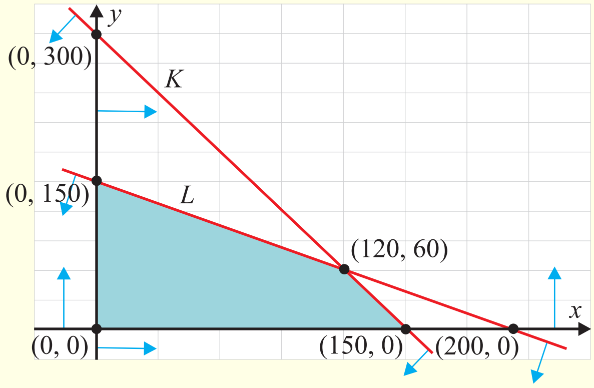


4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 150) and (150, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$\begin{aligned} 2x + y &= 300 \ldots(\mathbf{1}) \quad (\times -4) \\ 3x + 4y &= 600 \ldots(\mathbf{2}) \end{aligned}$$

$$\begin{aligned} -8x - 4y &= -1200 \\ 3x + 4y &= 600 \\ \hline -5x \quad &= -600 \Rightarrow x = 120 \end{aligned}$$

Substitute x = 120 back into Eqn. (**1**).

$\Rightarrow 2(120) + y = 300 \Rightarrow y = 300 - 240 = 60$

Therefore (120, 60) is the final vertex of the region.

5. Income = 11x + 10y is the function to be maximised.

	11x + 10y	**Income**
(0, 0)	11(0) + 10(0)	€0
(0, 150)	11(0) + 10(150)	€1500
(120, 60)	11(120) + 10(60)	€**1920**
(150, 0)	11(150) + 10(0)	€1650

Therefore, 120 videos and 60 DVDs give the maximum income.

Answers

11 (b) (i) $2x + y \le 300$, $3x + 4y \le 600$

11 (b) (ii) x = 120, y = 60

11 (b) (iii)

Profit on videos = €11 – €6 = €5

Profit on DVDs = €10 – €8 = €2

Profit = 5x + 3y is the function to be maximised.

	5x + 2y	**Income**
(0, 0)	5(0) + 2(0)	€0
(0, 150)	5(0) + 2(150)	€300
(120, 60)	5(120) + 2(60)	€720
(150, 0)	5(150) + 2(0)	€**750**

Therefore, 150 videos and 0 DVDs will maximise profit.

2003

11 (b) A developer is planning a scheme of holiday homes, consisting of large and small bungalows. Each large bungalow will accommodate 8 people and each small bungalow will accommodate 6 people. The development is not permitted to accommodate more than 216 people. The floor area of each large bungalow is 200 m^2 and the floor area of each small bungalow is 100 m^2. The total floor area of all the bungalows must not exceed 4000 m^2.

(i) Taking x as the number of large bungalows and y as the number of small bungalows, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The expected net annual income from each large bungalow is €14 000 and from each small bungalow is €8000. How many of each type should be built in order to maximise the total expected net annual income?

(iii) The developer decides to build as indicated in part **(ii)**. The cost of building each large bungalow is €110 000 and the cost of building each small bungalow is €85 000. The total cost of the development is equal to the building costs plus €1.58 million. How many years will it take to recoup the total cost of the development?

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of large bungalows
Let y = Number of small bungalows

2.

	Large bungalows	Small bungalows	Restriction
Accommodation	$8x$	$6y$	216
Floor Space	$200x$	$100y$	4000

Accommodation inequality: $8x+6y\le 216 \Rightarrow 4x+3y\le 108$

Floor space inequality: $200x+100y\le 4000 \Rightarrow 2x+y\le 40$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

CONT......

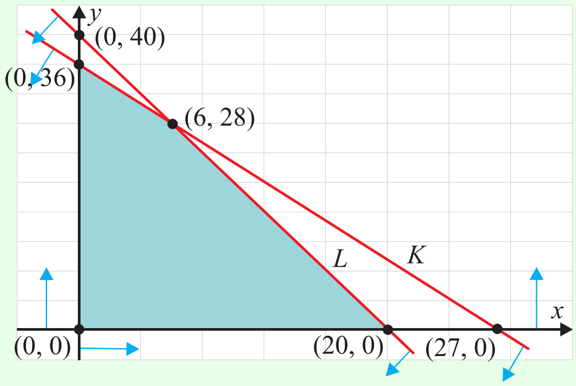
3. Plot the four inequalities.

Graph $4x+3y\le 108$. Draw the line 4*x* + 3*y* = 108. Call it *K*.

Intercepts: (0, 36), (27, 0). Test with (0, 0) $\Rightarrow 4(0)+3(0)=0\le 108$. This is true. Shade the side of the line that contains (0, 0).

Graph $2x+y\le 40$. Draw the line 2*x* + *y* = 40. Call it *L*.

Intercepts: (0, 40), (20, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0\le 40$. This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 36) and (20, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$4x+3y=108 \ldots\mathbf{(1)}$$
$$2x+y=40 \ldots\mathbf{(2)}\quad(\times -3)$$

$$\begin{aligned} 4x+3y &= 108\\ -6x-3y &= -120\\ \hline -2x &= -12 \Rightarrow x=6 \end{aligned}$$

Substitute *x* = 6 back into Eqn. (**2**).

$\Rightarrow 2(6)+y=40 \Rightarrow y=40-12=28$

Therefore (6, 28) is the final vertex of the region.

5. Income = 14000*x* + 8000*y* is the function to be maximised.

	14000*x* + 8000*y*	**Income**
(0, 0)	14000(0) + 8000(0)	€0
(0, 36)	14000(0) + 8000(36)	€288,000
(6, 28)	14000(6) + 8000(28)	€**308,000**
(20, 0)	14000(20) + 8000(0)	€280,000

Therefore, 6 large bungalows and 28 small bungalows give the maximum rental income.

ANSWERS

11 (b) (i) $4x+3y\le 108,\ 2x+y\le 40$

11 (b) (ii) *x* = 6, *y* = 28

11 (b) (iii)

Cost of development $=6\times €110000+28\times €85000+€1580000=€4,620,000$

No. of years $=\dfrac{€4,620,000}{€308,000}=15$

2002

11 (b) A new ship is being designed. It can have two types of cabin accommodation for passengers — type A cabins and type B cabins.

Each type A cabin accommodates 6 passsengers and each type B cabin accommodates 3 passengers. The maximum number of passengers that the ship can accommodate is 330.

Each type A cabin occupies 50 m^3 of floor space. Each type B cabin occupies 10 m^3 of floor space. The total amount of floor space occupied by cabins cannot exceed 2300 m^3.

(i) Taking x to represent the number of type A cabins and y to represent the number of type B cabins, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The income on each voyage from renting the cabins to passengers is €600 for each type A cabin and €180 for each type B cabin. How many of each type of cabin should the ship have so as to maximise income, assuming that all cabins are rented?

(iii) What is the maximum possible income on each voyage from renting the cabins?

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of Type A cabins
Let y = Number of Type B cabins

2.

	Type A	Type B	Restriction
Accommodation	$6x$	$3y$	330
Floor space	$50x$	$10y$	2300

Accommodation inequality: $6x+3y \le 330 \Rightarrow 2x+y \le 110$

Floor space inequality: $50x+10y \le 2300 \Rightarrow 5x+y \le 230$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.

Graph $2x+y \le 110$. Draw the line $2x + y = 110$. Call it K.

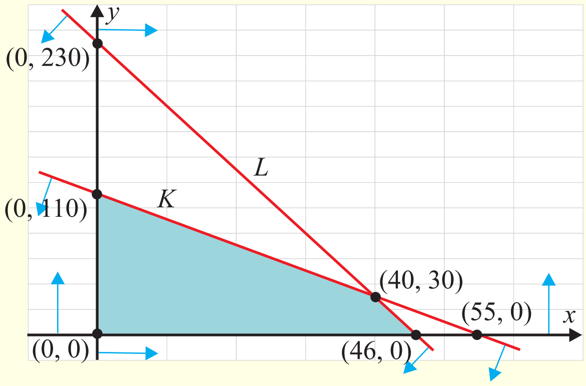
Intercepts: (0, 110), (55, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0 \le 110$. This is true.

Shade the side of the line that contains (0, 0).

CONT......

Graph $5x + y \leq 230$. Draw the line $5x + y = 230$. Call it L.

Intercepts: (0, 230), (46, 0). Test with (0, 0) $\Rightarrow 5(0)+(0)=0 \leq 230$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 110) and (46, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x + y = 110 \ldots\mathbf{(1)} \quad (\times -1)$$
$$5x + y = 230 \ldots\mathbf{(2)}$$

$$\begin{array}{rl} -2x - y &= -110 \\ 5x + y &= 230 \\ \hline 3x &= 120 \Rightarrow x = 40 \end{array}$$

Substitute x = 40 back into Eqn. (**1**).
$\Rightarrow 2(40) + y = 110 \Rightarrow y = 110 - 80 = 30$
Therefore (40, 30) is the final vertex of the region.

5. Income = 600x + 180y is the function to be maximised.

	600x + 180y	**Income**
(0, 0)	600(0) + 180(0)	€0
(0, 110)	600(0) + 180(110)	€19,800
(40, 30)	600(40) + 180(30)	€**29,400**
(46, 0)	600(46) + 180(0)	€27,600

Therefore, 40 type A cabins and 30 type B cabins give the maximum rental income.

Answers

11 (b) (i) $2x + y \leq 110$, $5x + y \leq 230$
11 (b) (ii) x = 40, y = 30
11 (b) (iii) €29,400

2001

11 (b) Houses are to be built on 9 hectares of land.
Two types of houses, bungalows and semi-detached houses, are possible.

Each bungalow occupies one fifth of a hectare.
Each semi-detached house occupies one tenth of a hectare.

The cost of building a bungalow is IR£80 000.
The cost of building a semi-detached house is IR£50 000.
The total cost of building the houses cannot be greater than IR£4 million.

(i) Taking x to represent the number of bungalows and y to represent the number of semi-detached houses, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The profit on each bungalow is IR£10 000. The profit on each semi-detached house is IR£7000. How many of each type of house should be built so as to maximise profit?

Solution

11 (b)

Maximising and Minimising Problems

Steps
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of bungalows
Let y = Number of semi-detached houses

2.

	Bungalows	Houses	Restriction
Space	$\frac{1}{5}x$	$\frac{1}{10}y$	9
Cost	$80000x$	$50000y$	4000000

Space inequality: $\frac{1}{5}x+\frac{1}{10}y\le 9 \Rightarrow 2x+y\le 90$

Cost inequality: $80000x+50000y\le 4000000 \Rightarrow 8x+5y\le 400$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

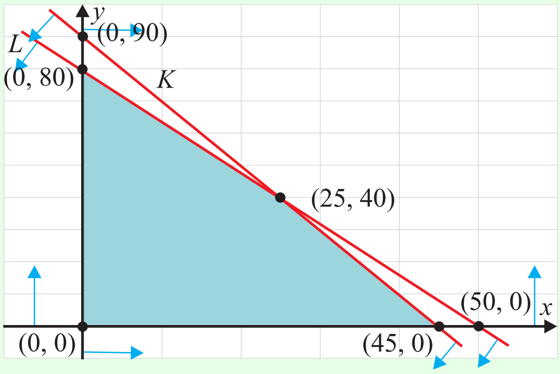
3. Plot the four inequalities.
Graph $2x+y\le 90$. Draw the line $2x + y = 90$. Call it K.

Intercepts: (0, 90), (45, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0\le 90$. This is true.
Shade the side of the line that contains (0, 0).

Graph $8x+5y\le 400$. Draw the line $8x + 5y = 400$. Call it L.

Intercepts: (0, 80), (50, 0). Test with (0, 0) $\Rightarrow 8(0)+5(0)=0\le 400$.
This is true. Shade the side of the line that contains (0, 0).

Cont......



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 80) and (45, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$2x + y = 90 \ldots\ldots(\mathbf{1}) \quad (\times -4)$$
$$8x + 5y = 400 \ldots(\mathbf{2})$$

$$-8x - 4y = -360$$
$$8x + 5y = 400$$
$$y = 40$$

Substitute $y = 40$ back into Eqn. (**1**).

$\Rightarrow 2x + (40) = 90 \Rightarrow 2x = 50 \Rightarrow x = 25$

Therefore (25, 40) is the final vertex of the region.

5. Profit = 10000*x* + 7000*y* is the function to be maximised.

	10000*x* + 7000*y*	**Income**
(0, 0)	10000(0) + 7000(0)	€0
(0, 80)	10000(0) + 7000(80)	€**560,000**
(25, 40)	10000(25) + 7000(40)	€530,000
(45, 0)	10000(45) + 7000(0)	€450,000

Therefore, 0 bungalows and 80 semi-detached houses give the maximum profit.

ANSWERS

11 (b) (i) $2x + y \le 90,\ 8x + 5y \le 400$

11 (b) (ii) $x = 0,\ y = 80$

2000

11 (b) Two types of machines, type A and type B, can be purchased for a new factory. Each machine of type A costs IR£1600. Each machine of type B costs IR£800. The purchase of the machines can cost, at most, IR£27,200.

Each machine of type A needs 90 m^2 of floor space in the factory. Each machine of type B needs 54 m^2 of floor space.

The maximum amount of floor space available for the machines is 1620 m^2.

(i) If x represents the number of machines of type A and y represents the number of machines of type B, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The daily income from the use of each machine of type A is IR£75. The daily income from the use of each machine of type B machine is IR£42. How many of each type of machine should be purchased so as to maximise daily income?

(iii) What is the maximum daily income?

Solution

11 (b)

Maximising and Minimising Problems

Steps
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of type A machines
Let y = Number of type B machines

2.

	Type A	Type B	Restriction
Cost	$1600x$	$800y$	27200
Floor space	$90x$	$54y$	1620

Cost inequality: $1600x+800y \le 27200 \Rightarrow 2x+y \le 34$

Floor space inequality: $90x+54y \le 1620 \Rightarrow 5x+3y \le 90$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

Cont......

3. Plot the four inequalities.

Graph $2x+y\le 34$. Draw the line $2x + y = 34$. Call it K.

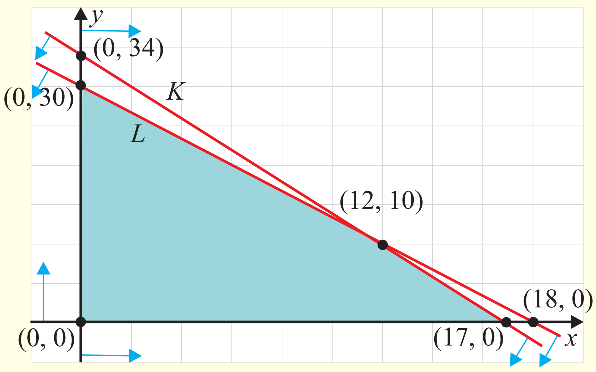
Intercepts: (0, 34), (17, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0\le 34$. This is true.

Shade the side of the line that contains (0, 0).

Graph $5x+3y\le 90$. Draw the line $5x + 3y = 90$. Call it L.

Intercepts: (0, 30), (18, 0). Test with (0, 0) $\Rightarrow 5(0)+3(0)=0\le 90$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (17, 0).

The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x+y=34\ldots(\mathbf{1})\quad(\times -3)$$
$$5x+3y=90\ldots(\mathbf{2})$$

$$-6x-3y=-102$$
$$5x+3y=90$$
$$-x=-12\Rightarrow x=12$$

Substitute $x = 12$ back into Eqn. (**1**).

$\Rightarrow 2(12)+y=34\Rightarrow y=34-24=10$

Therefore (12, 10) is the final vertex of the region.

5. Income = $75x + 42y$ is the function to be maximised.

	$75x + 42y$	**Income**
(0, 0)	75(0) + 42(0)	€0
(0, 30)	75(0) + 42(30)	€1260
(12, 10)	75(12) + 42(10)	€**1320**
(17, 0)	75(17) + 42(0)	€1275

Therefore, 12 type A machines and 10 type B machines give the maximum income.

Answers

11 (b) (i) $2x+y\le 34,\ 5x+3y\le 90$

11 (b) (ii) A = 12, B = 10

11 (b) (iii) £1320

1999

11 (b) A company uses small trucks and large trucks to transport its products in crates. The crates are all of the same size.

On a certain day 10 truck drivers at most are available. Each truck requires one driver only.

Small trucks take 10 minutes each to load and large trucks take 30 minutes each to load. The total loading time must not be more than 3 hours. Only one truck can be loaded at a time.

(i) If x represents the number of small trucks used and y represents the number of large trucks used, write down two inequalities in x and y.

Illustrate these on graph paper.

(ii) Each small truck carries 30 crates and each large truck carries 70 crates. How many of each type of truck should be used to maximize the number of crates to be transported that day?

SOLUTION

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of small trucks
Let y = Number of large trucks

2.

	Small trucks	Large trucks	Restriction
Drivers	x	y	10
Loading time	$10x$	$30y$	180

Drivers inequality: $x + y \le 10$

Loading time inequality: $10x + 30y \le 180 \Rightarrow x + 3y \le 18$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.

Graph $x + y \le 10$. Draw the line $x + y = 10$. Call it K.

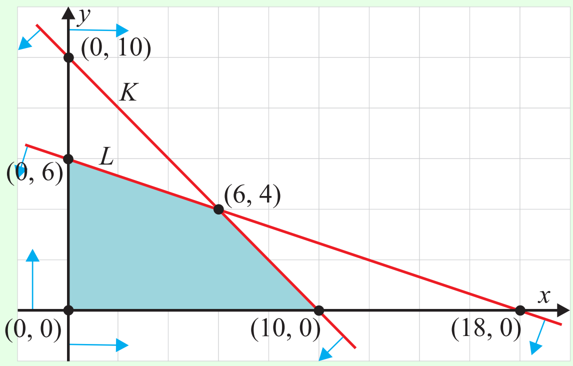
Intercepts: (0, 10), (10, 0). Test with (0, 0) $\Rightarrow (0) + (0) = 0 \le 10$. This is true.
Shade the side of the line that contains (0, 0).

CONT......

Graph $x+3y \le 18$. Draw the line $x + 3y = 18$. Call it L.

Intercepts: (0, 6), (18, 0). Test with (0, 0) $\Rightarrow (0)+3(0)=0\le 18$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 6) and (10, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$x+y\ \ =10...(\mathbf{1})\ \ (\times -1)$$
$$x+3y=18...(\mathbf{2})$$

$$-x-y\ \ =-10$$
$$x+3y=18$$
$$2y=8\Rightarrow y=4$$

Substitute $y = 4$ back into Eqn. (**1**).

$\Rightarrow x+(4)\ \ =10 \Rightarrow x=6$

Therefore (6, 4) is the final vertex of the region.

5. Crates cargo = $30x + 70y$ is the function to be maximised.

	$30x + 70y$	**Cargo**
(0, 0)	30(0) + 70(0)	0
(0, 6)	30(0) + 70(6)	420
(6, 4)	30(6) + 70(4)	**460**
(10, 0)	30(10) + 70(0)	300

Therefore, 6 small trucks and 4 large trucks give the maximum cargo.

ANSWERS:

11 (b) (i) $x+y\le 10,\ x+3y\le 18$

11 (b) (ii) $x = 6, y = 4$

1998

11 (b) A company produces two products, A and B.

Each unit of the two products must be processed on two assembly lines, the red line and the blue line, for a certain length of time.

Each unit of A requires 3 hours on the red line and 1 hour on the blue line.
Each unit of B requires 1 hour on the red line and 2 hours on the blue line.

Each week, the maximum time available on the red line is 60 hours and the maximum time available on the blue line is 40 hours.

(i) If x represents the number of units of A produced in a week and y represents the number of units of B produced in a week, write down two inequalities in x and y. Illustrate these on graph paper.

(ii) The profit made on each unit of A is twice the profit made on each unit of B. How many units of each product must be manufactured in a week so as to maximise the profit?

(iii) If the maximum profit that can be made in a week is IR£1980, calculate the profit made on each unit of A and on each unit of B.

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of units of A
Let y = Number of units of B

2.

	A	B	Restriction
Red line	$3x$	y	60
Blue line	x	$2y$	40

Red line inequality: $3x + y \leq 60$

Blue line inequality: $x + 2y \leq 40$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

3. Plot the four inequalities.

Graph $3x + y \leq 60$. Draw the line $3x + y = 60$. Call it K.

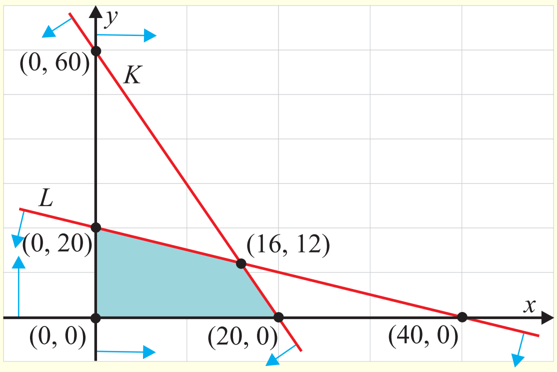
Intercepts: (0, 60), (20, 0). Test with (0, 0) $\Rightarrow 3(0) + (0) = 0 \leq 60$. This is true.
Shade the side of the line that contains (0, 0).

CONT......

Graph $x+2y \leq 40$. Draw the line $x + 2y = 40$. Call it L.

Intercepts: (0, 20), (40, 0). Test with (0, 0) $\Rightarrow (0)+2(0)=0 \leq 40$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 20) and (20, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$3x+y=60 \ldots\mathbf{(1)} \quad (\times -2)$$
$$x+2y=40 \ldots\mathbf{(2)}$$

$$-6x-2y=-120$$
$$x+2y=40$$
$$-5x = -80 \Rightarrow x=16$$

Substitute $x = 16$ back into Eqn. (**2**).

$\Rightarrow (16)+2y=40 \Rightarrow 2y=24 \Rightarrow y=12$

Therefore (16, 12) is the final vertex of the region.

5. Profit = $2x + y$ is the function to be maximised.

	$2x + y$	**Profit**
(0, 0)	2(0) + (0)	0
(0, 20)	2(0) + (20)	20
(16, 12)	2(16) + (12)	**44**
(20, 0)	2(20) + (0)	40

Therefore, 16 units of A and 12 units of B give the maximum profit.

Answers:

11 (b) (i) $3x+y \leq 60,\ x+2y \leq 40$

11 (b) (ii) 16 of A and 12 of B

11 (b) (iii)

A profit of £44 was generated from producing 16 units of A and 12 units of B using the fact that £2 profit was made of each unit of A against £1 profit for each unit of B.

Profit on each unit of B $= \dfrac{£1980}{44} = £45$

Profit on each unit of A $= £90$

1997

11 (b) A factory, which manufactures television sets makes two types of set - a wide screen model and a standard model.

In any week, 500 sets at most can be manufactured.

Each wide screen model costs IR£200 to produce. Each standard model costs IR£150 to produce. Total weekly production costs must not be greater than IR£90,000.

(i) If the factory manufactures *x* of the wide screen model and *y* of the standard model, write down two inequalities in *x* and *y* and illustrate these on graph paper.

(ii) If the profit on a wide screen model is IR£100 and the profit on a standard model is IR£70, how many of each type of set should be manufactured in order to maximise profit?

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS

1. Choose two variables *x* and *y* to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of wide screen models
Let y = Number of standard models

2.

	Wide screen	Standard	Restriction
Output	x	y	500
Cost	$200x$	$150y$	90000

Output inequality: $x + y \le 500$

Costs inequality: $200x + 150y \le 90000 \Rightarrow 4x + 3y \le 1800$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.

Graph $x + y \le 500$. Draw the line $x + y = 500$. Call it K.

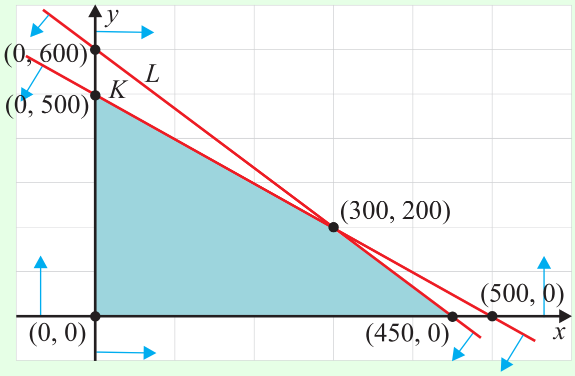
Intercepts: (0, 500), (500, 0). Test with (0, 0) $\Rightarrow (0) + (0) = 0 \le 500$. This is true.
Shade the side of the line that contains (0, 0).

CONT......

Graph $4x+3y\leq 1800$. Draw the line $4x + 3y = 1800$. Call it L.

Intercepts: (0, 600), (450, 0). Test with (0, 0) $\Rightarrow 4(0)+3(0)=0\leq 1800$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 500) and (450, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$\begin{aligned} x+y &= 500 \ldots\ldots(\mathbf{1}) \quad (\times -3) \\ 4x+3y &= 1800 \ldots(\mathbf{2}) \end{aligned}$$

$$\begin{aligned} -3x-3y &= -1500 \\ 4x+3y &= 1800 \\ \hline x &= 300 \end{aligned}$$

Substitute $x = 300$ back into Eqn. (**1**).

$\Rightarrow (300)+y=500 \Rightarrow y=200$

Therefore (300, 200) is the final vertex of the region.

5. Profit = $100x + 70y$ is the function to be maximised.

	$100x + 70y$	**Profit**
(0, 0)	100(0) + 70(0)	£0
(0, 500)	100(0) + 70(500)	£35,000
(300, 200)	100(300) + 70(200)	£44,000
(450, 0)	100(450) + 70(0)	£45,000

Therefore, 450 wide screen models and 0 standard models give the maximum profit.

ANSWERS

11 (b) (i) $x+y\leq 500,\ 4x+3y\leq 1800$

11 (b) (ii) $x = 450,\ y = 0$

1996

11 (b) A property developer wishes to construct a business centre consisting of shops and offices. The floor space required for each shop is 60 m^2 and for each office is 20 m^2. The total floor space for the business centre cannot exceed 960 m^2.

The construction of each shop takes 5 working days to complete and each office 3 working days to complete. The developer has at most 120 working days to complete the construction.

(i) If the developer constructs x shops and y offices, write two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental charge is IR£200 per m^2 for a shop and IR£140 per m^2 for an office, how many of each type should be built so as to maximize the developer's rental income? Find this maximum rental income.

(iii) If each shop provides 7 jobs and each office 3 jobs, write an expression in x and y for the total number of jobs to be provided. How many of each type should be built so as to maximize the number of jobs?

SOLUTION

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of shops
Let y = Number of offices

2.

	Shops	Offices	Restriction
Floor space (m^2)	$60x$	$20y$	960
Time (Days)	$5x$	$3y$	120

Adults inequality: $60x + 20y \leq 960 \Rightarrow 3x + y \leq 48$

Children inequality: $5x + 3y \leq 120$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

CONT......

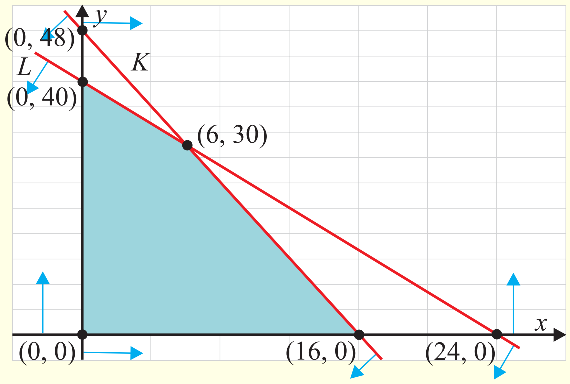
3. Plot the four inequalities.

Graph $3x + y \le 48$. Draw the line $3x + y = 48$. Call it K.

Intercepts: (0, 48), (16, 0). Test with (0, 0) $\Rightarrow 3(0) + (0) = 0 \le 48$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x + 3y \le 120$. Draw the line $5x + 3y = 120$. Call it L.

Intercepts: (0, 40), (24, 0). Test with (0, 0) $\Rightarrow 5(0) + 3(0) = 0 \le 120$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 40) and (16, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$3x + y = 48 \ldots\ldots(\mathbf{1}) \quad (\times -3)$$
$$5x + 3y = 120 \ldots(\mathbf{2})$$

$$-9x - 3y = -144$$
$$5x + 3y = 120$$
$$-4x = -24 \Rightarrow x = 6$$

Substitute $x = 6$ back into Eqn. (**1**).

$\Rightarrow 3(6) + y = 48 \Rightarrow y = 48 - 18 = 20$

Therefore (6, 20) is the final vertex of the region.

5. Rental income = $200x + 140y$ is the function to be maximised.

	$200x + 140y$	**Income**
(0, 0)	200(0) + 140(0)	£0
(0, 40)	200(0) + 140(40)	£**5600**
(6, 20)	200(6) + 140(20)	£4000
(16, 0)	200(16) + 140(0)	£3200

Therefore, 0 shops and 40 offices give the maximum rental income.

5. No. of jobs = $7x + 3y$ is the function to be maximised.

	$7x + 3y$	**Jobs**
(0, 0)	7(0) + 3(0)	0
(0, 40)	7(0) + 3(40)	**120**
(6, 20)	7(6) + 3(20)	58
(16, 0)	7(16) + 3(0)	112

Therefore, 0 shops and 40 offices give the maximum number of jobs.