LINEAR PROGRAMMING (Q 11, PAPER 2)

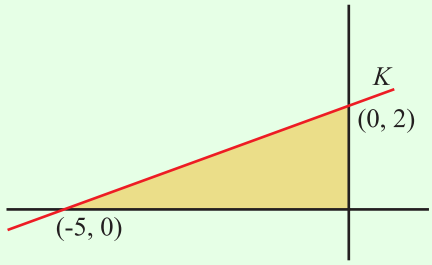
LESSON NO. 2: FINDING INEQUALITIES FROM A DIAGRAM

2007

11 (a) The line K cuts the x-axis at $(-5, 0)$ and the y-axis at $(0, 2)$.

(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.



SOLUTION

11 (a) (i)

$(-5, 0)$ → x_1, y_1 $(0, 2)$ → x_2, y_2

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **3**

Equation of a line: $y - y_1 = m(x - x_1)$ **4**

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{0-(-5)} = \dfrac{2}{5}$

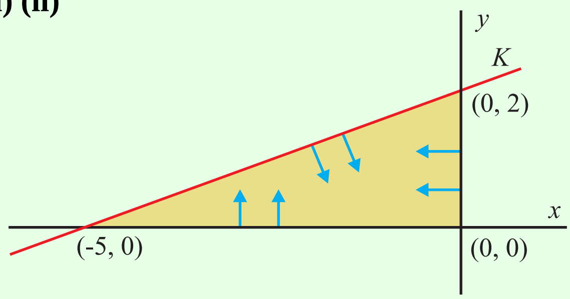
Equation of line K: $y - 0 = \frac{2}{5}(x-(-5)) \Rightarrow y = \frac{2}{5}(x+5)$

$\Rightarrow 5y = 2(x+5)$

$\Rightarrow 5y = 2x+10$

$\therefore 2x - 5y + 10 = 0$

11 (a) (ii)



VERTICAL LINES:	$\geq$: → (Right) $\leq$: ← (Left)
HORIZONTAL LINES:	$\geq$: ↑ (Above) $\leq$: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Left of the y-axis $\Rightarrow y \leq 0$

STEPS

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow 2(0) - 5(0) + 10 = 10 \geq 0$

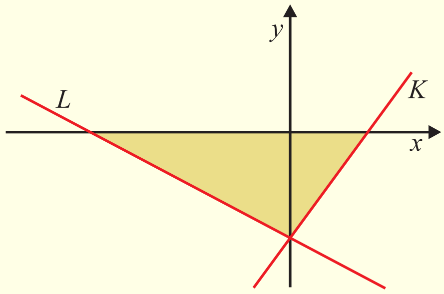
3. The indicated region is on the same side as (0, 0).

Therefore, $2x - 5y + 10 \geq 0$ is the inequality of the indicated region.

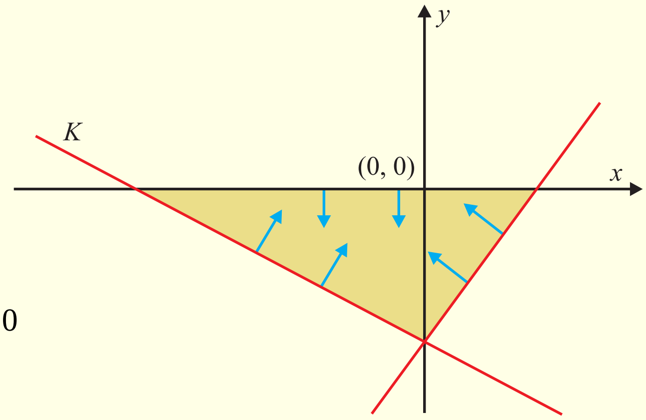
Three inequalities: $x \leq 0,\ y \geq 0,\ 2x - 5y + 10 \geq 0$

2006

11 (a) The equation of the line L is $5x + 8y + 40 = 0$.
The equation of the line K is $10x - 7y - 35 = 0$.
Write down the 3 inequalities that together define the shaded region in the diagram.



SOLUTION



Equation of L: $5x + 8y + 40 = 0$

Test with (0, 0): $5(0)+8(0)+40 = 40 \geq 0$

Shaded side is on the same side as (0, 0).

$\therefore 5x+8y+40 \geq 0$

Equation of K: $10x - 7y - 35 = 0$

Test with (0, 0): $10(0)-7(0)-35 = -35 \leq 0$

Shaded side is on the same side as (0, 0).

$\therefore 10x - 7y - 35 \leq 0$

Below the x-axis: $y = 0$
Shaded side under this line.
$\therefore y \leq 0$

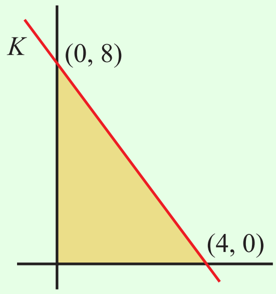
HORIZONTAL LINES: $\geq$: ↑ (Above)
$\leq$: ↓ (Below)

2005

11 (a) The line K cuts the x-axis at (4, 0) and the y-axis at (0, 8).

(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.



SOLUTION

11 (a) (i)

(0, 8)	(4, 0)
↓ ↓	↓ ↓
x_1 y_1	x_2 y_2

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ **3**

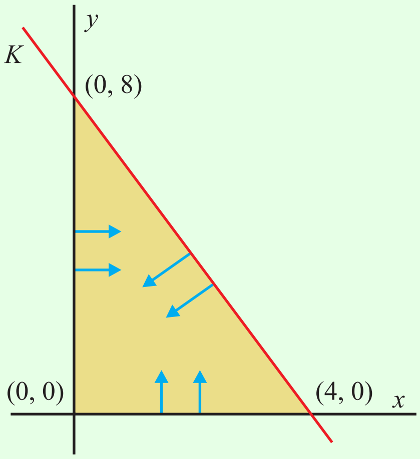
Equation of a line: $y - y_1 = m(x - x_1)$ **4**

Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-8}{4-0} = \frac{-8}{4} = -2$

Equation of line K: $y - 8 = -2(x - 0) \Rightarrow y - 8 = -2x$

$\therefore 2x + y - 6 = 0$

11 (a) (ii)



VERTICAL LINES: $\geq$: → (Right) $\leq$: ← (Left)

HORIZONTAL LINES: $\geq$: ↑ (Above) $\leq$: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y \geq 0$

> **STEPS**
> **2.** Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.
> **3.** The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow 2(0) + (0) - 8 = -8 \leq 0$

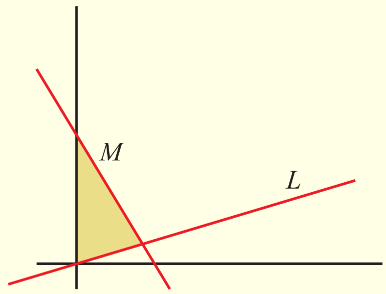
3. The indicated region is on the same side as (0, 0).

Therefore, $2x + y - 8 \leq 0$ is the inequality of the indicated region.

Three inequalities: $x \geq 0,\ y \geq 0,\ 2x + y - 8 \leq 0$

2004

11 (a) The equation of the line L is $x-2y=0$.
The equation of the line M is $2x+y=4$.
Write down the three inequalities that together define the shaded region in the diagram.



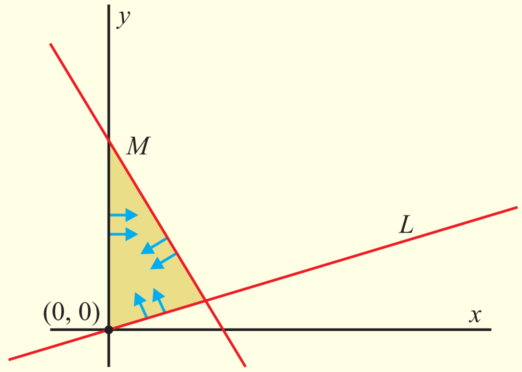
SOLUTION

Equation of L: $x-2y=0$

Test with (0, 1): $(0)-2(1)=-2\le 0$

Shaded side is on the same side as (0, 1).

$\therefore x-2y\le 0$



Equation of M: $2x+y=4$

Test with (0, 0): $2(0)+(0)=0\le 4$

Shaded side is on the same side as (0, 0).

$\therefore 2x+y\le 4$

Right of the y-axis: $x=0$

Shaded side under this line.

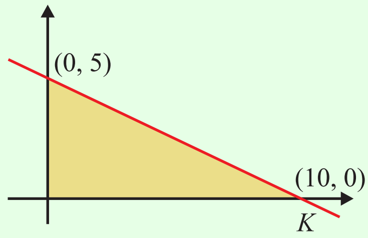
$\therefore x\ge 0$

VERTICAL LINES: $\ge$: $\rightarrow$ (Right)
$\le$: $\leftarrow$ (Left)

ANSWER: $x-2y\le 0,\ x\ge 0,\ 2x+y\le 4$

2003

11 (a) The line K cuts the x-axis at (10, 0) and the y-axis at (0, 5).



(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.

SOLUTION

11 (a) (i)

(0, 5)	(10, 0)
↓ ↓	↓ ↓
x_1 y_1	x_2 y_2

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ **3**

Equation of a line: $y - y_1 = m(x - x_1)$ **4**

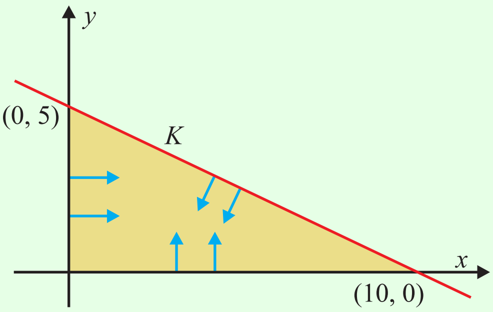
Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{10-0} = \frac{-5}{10} = -\frac{1}{2}$

Equation of line K: $y - 5 = -\frac{1}{2}(x - 0) \Rightarrow 2(y-5) = -x$

$\Rightarrow 2y - 10 = -x$

$\therefore x + 2y - 10 = 0$

11 (a) (ii)



VERTICAL LINES:	$\geq$: → (Right) $\leq$: ← (Left)

HORIZONTAL LINES:	$\geq$: ↑ (Above) $\leq$: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y \geq 0$

STEPS

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow (0) + 2(0) - 10 = -10 \leq 0$

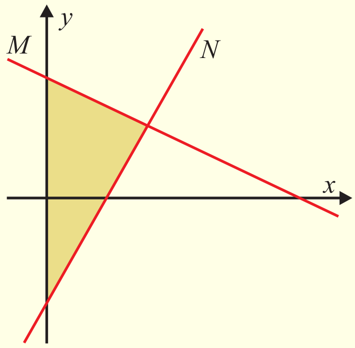
3. The indicated region is on the same side as (0, 0).

Therefore, $x + 2y - 10 \leq 0$ is the inequality of the indicated region.

Three inequalities: $x + 2y - 10 \leq 0,\ y \geq 0,\ x \geq 0$

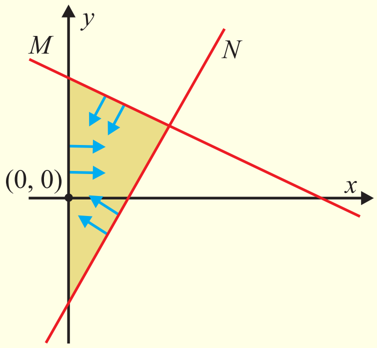
2002

11 (a) The equation of the line M is $2x + y = 10$.
The equation of the line N is $4x - y = 8$.



Write down the three inequalities that define the shaded region in the diagram.

SOLUTION



Equation of M: $2x + y = 10$

Test with (0, 0): $2(0)+(0)=0\leq 10$

Shaded side is on the same side as (0, 0).

$\therefore 2x+y\leq 10$

Equation of N: $4x-y=8$

Test with (0, 0): $4(0)-(0)=0\leq 8$

Shaded side is on the same side as (0, 0).

$\therefore 4x-y\leq 8$

y-axis: $x = 0$

Shaded side is right of this line.

$\therefore x\geq 0$

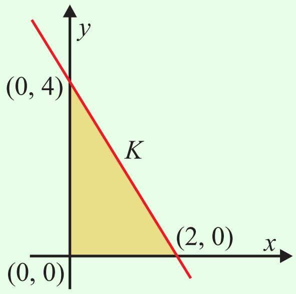
VERTICAL LINES: $\geq$: $\rightarrow$ (Right)
$\leq$: $\leftarrow$ (Left)

2000

11 (a) The line K passes through the points (2, 0) and (0, 4).

(i) Find the equation of the line K.

(ii) Write down three inequalities which define the shaded region in the diagram.



SOLUTION

11 (a) (i)

(2, 0)	(0, 4)
↓ ↓	↓ ↓
x_1 y_1	x_2 y_2

Slope: $m=\frac{y_2-y_1}{x_2-x_1}$ 3

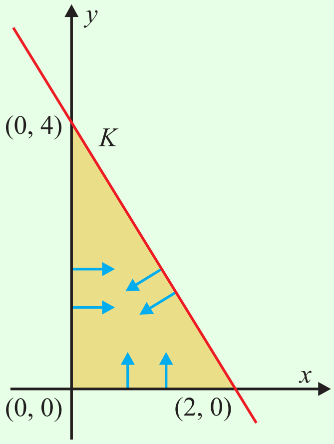
Equation of a line: $y-y_1=m(x-x_1)$ 4

Slope $m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-0}{0-2}=\frac{4}{-2}=-2$

Equation of line K: $y-0=-2(x-2)\Rightarrow y=-2x+4$

$\therefore 2x+y=4$

11 (a) (ii)



VERTICAL LINES: $\geq$: → (Right) $\leq$: ← (Left)

HORIZONTAL LINES: $\geq$: ↑ (Above) $\leq$: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x\geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y\geq 0$

STEPS
2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.
3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K\Rightarrow 2(0)+(0)=0\leq 4$

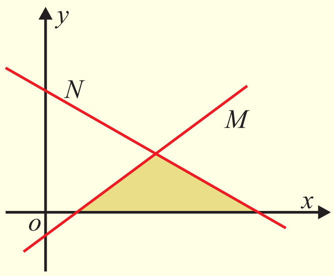
3. The indicated region is on the same side as (0, 0).

Therefore, $2x+y\leq 4$ is the inequality of the indicated region.

Three inequalities: $2x+y\leq 4,\ x\geq 0,\ y\geq 0$

1999

11 (a) The equation of the line M is $x - y - 1 = 0$ and the equation of the line N is $x + 2y - 6 = 0$. Write down the three inequalities which define the triangular region indicated in the diagram.



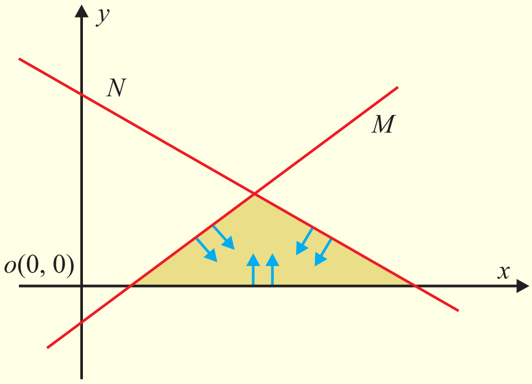
SOLUTION

Equation of M: $x - y - 1 = 0$

Test with (0, 0): $(0) - (0) - 1 = -1 \le 0$

Shaded side is on the opposite side as (0, 0).

$\therefore x - y - 1 \ge 0$



Equation of N: $x + 2y - 6 = 0$

Test with (0, 0): $(0) + 2(0) - 6 = -6 \le 0$

Shaded side is on the same side as (0, 0).

$x + 2y - 6 \le 0$

Above the x-axis: $y = 0$

Shaded side above this line.

$\therefore y \ge 0$

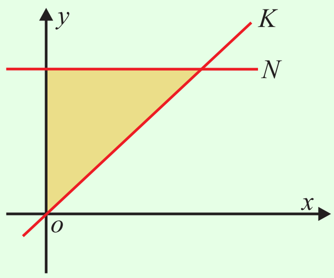
HORIZONTAL LINES:	$\ge$: ↑ (Above) $\le$: ↓ (Below)

1996

11 (a) The equation of the line K is $y - x = 0$ and the equation of the line N is $y - 4 = 0$.

(i) Write down the three inequalities which define the triangular region indicated in the diagram.

(ii) In a diagram, illustrate the set of points (x, y) that satisfy $y - 4 \ge 0$, $y - x \le 0$ and $x - 6 \le 0$.



SOLUTION

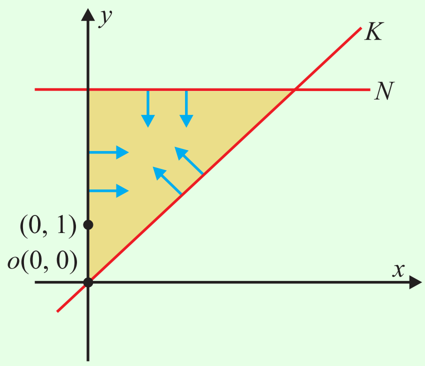
11 (a) (i)

Equation of K: $y - x = 0$

Test with (0, 1): $(1) - (0) = 1 \ge 0$

Shaded side is on the same side as (0, 1).

$\therefore y - x \ge 0$



CONT.....

Equation of *N*: $y-4=0$

Test with (0, 0): $(0)-4=-4\leq 0$

Shaded side is on the same side as (0, 0).

$\therefore y-4\leq 0$

y-axis: *x* = 0

Shaded side is right of the line.

$\therefore x\geq 0$

VERTICAL LINES: $\geq$: → (Right) $\leq$: ← (Left)

11 (a) (ii)

Graph $y-4\geq 0$.

Draw $y-4=0\Rightarrow y=4$.

Draw a line through *y* = 4 and shade above the line.

HORIZONTAL LINES: $\geq$: ↑ (Above) $\leq$: ↓ (Below)

Graph $y-x\leq 0$.

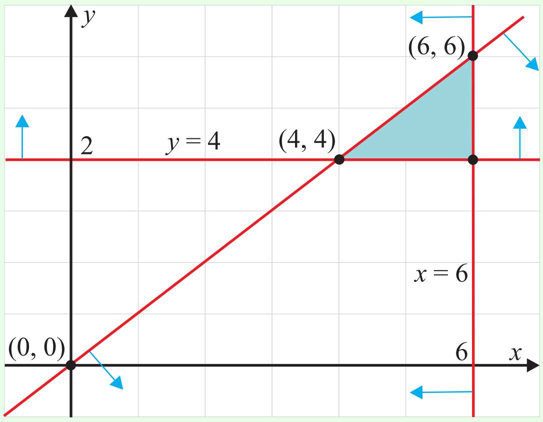
Draw *y* = *x*. This is a line through (0, 0). It contains points where the first and second co-ordinates are equal like (4, 4) and (6, 6).

Graph $x-6\leq 0$.

Draw $x-6=0\Rightarrow x=6$.

Draw a line through *x* = 6 and shade to the left of the line.

VERTICAL LINES: $\geq$: → (Right) $\leq$: ← (Left)



Draw the lines. The blue arrows indicate the side of the line for which the inequality is true. These regions all overlap in the region where the three lines intersect. Shade in this region. The points in this region simultaneously satisfy the three inequalities.