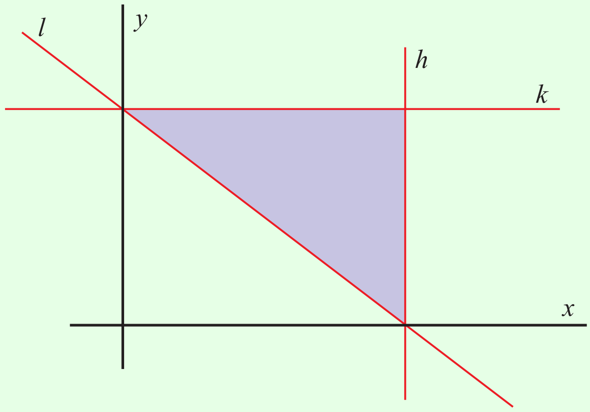
LINEAR PROGRAMMING (Q 11, PAPER 2)

2011

11. (a) The diagram shows the lines
l: $2x + 3y - 6 = 0$,
h: $x - 3 = 0$ and k: $y - 2 = 0$.
Write down the three inequalities that together define the shaded region in the diagram.



(b) A garage is starting a van rental business. The garage will rent out two types of vans, small vans and large vans.

To set up the business, each small van costs €20 000 and each large van costs €40 000. The garage has at most €800 000 to purchase the vans.

Each small van requires 18 m^2 of parking space and each large van requires 24 m^2 of parking space. The garage has at most 576 m^2 of parking space available for the vans.

(i) Taking x as the number of small vans and y as the number of large vans, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The garage charges €40 a day to rent a small van and €50 a day to rent a large van. How many of each should the garage rent to maximise rental income, assuming that all vans are rented.

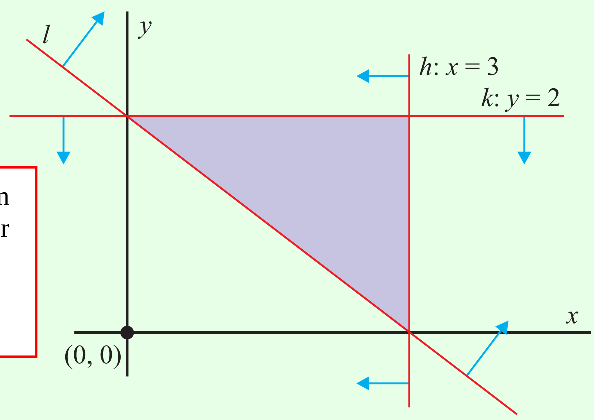
(iii) The garage incurs daily expenses of €12 for each van. Calculate the maximum daily profit from renting the vans.

SOLUTION

11 (a)

Shaded area is the the left of line h: $x \le 3$

Shaded area is below line k: $y \le 2$



> Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.
> The side of the line with (0, 0) obeys the inequality.
> The other side is the opposite to the inequality.

$l: \quad 2(0)+3(0)-6$

$= 0+0-6$

$= -6 \le 0$

$l: 2x+3y-6 \ge 0$

11 (b)

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of small vans
Let y = Number of large vans

2.

	Small vans	Large vans	Restriction
Cost	$20000x$	$40000y$	800000
Area	$18x$	$24y$	576

Cost inequality: $20000x + 40000y \leq 800000 \Rightarrow x + 2y \leq 40$

Area inequality: $18x + 24y \leq 576 \Rightarrow 3x + 2y \leq 96$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

3. Plot the four inequalities.

Graph $x + 2y \leq 40$. Draw the line $x + 2y = 40$. Call it k.

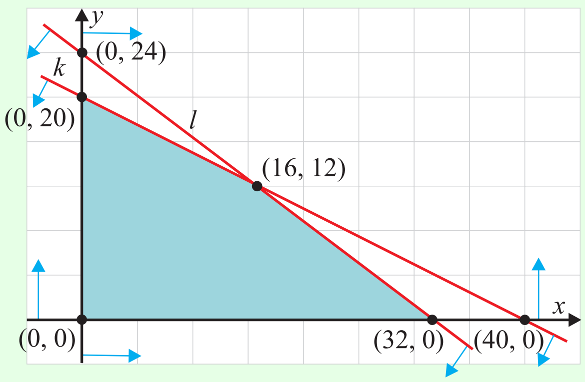
Intercepts: (0, 20), (40, 0). Test with (0, 0) $\Rightarrow 0 + 2(0) = 0 \leq 40$. This is true.

Shade the side of the line that contains (0, 0).

Graph $3x + 4y \leq 96$. Draw the line $3x + 4y = 96$. Call it l.

Intercepts: (0, 24), (32, 0). Test with (0, 0) $\Rightarrow 3(0) + 4(0) = 0 \leq 96$.

This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 20) and (32, 0).
The only one you need to work out simultaneously is where the lines *k* and *l* intersect.

$$x + 2y = 40......(\mathbf{1})(\times -2)$$
$$3x + 4y = 96....(\mathbf{2})$$

$$-2x - 4y = -80$$
$$3x + 4y = 96$$
$$x \quad = 16$$

Substitute $y = 15$ back into Eqn. (**1**).

$(16) + 2y = 40 \Rightarrow 2y = 24 \Rightarrow y = 12$

Therefore (16, 12) is the final vertex of the region.

5. Rental Income = $40x + 50y$ is the function to be maximised.

	$40x + 50y$	**Income**
(0, 0)	40(0) + 50(0)	€0
(0, 20)	40(0) + 50(20)	€1000
(16, 12)	40(16) + 50(12)	€1240
(32, 0)	40(32) + 50(0)	€**1280**

Therefore, 32 small vans and 0 large vans give the maximum rental income.
The maximum rental income is €1280.

11 (b) (i) Graph drawn

11 (b) (ii) 32 small vans and 0 large vans give the maximum rental income.

11 (b) (iii) Maximum rental income is €1280.

Expenses = $32 \times €12 = €384$

Maximum daily profit = $€1280 - €384 = €896$