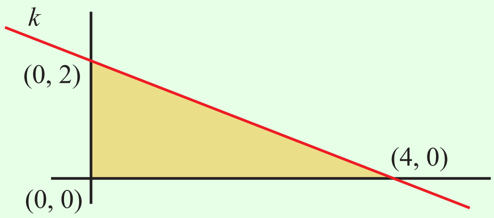
LINEAR PROGRAMMING (Q 11, PAPER 2)

2010

11 (a) The line k passes through the points (0, 2) and (4, 0).

(i) Find the equation of k.

(ii) Write down the three inequalities which define the shaded region in the diagram.



(b) A contractor has the task of loading containers onto a truck. There are two types of container: heavy containers which weigh 160 kg each and light containers which weigh 40 kg each. The truck can carry, at most, a total weight of 2080 kg.

The time taken to load a heavy container is 3 minutes. The time taken to load a light container is 2 minutes. The total time spent loading a truck cannot be greater than 54 minutes.

(i) Taking x as the number of heavy containers and y as the number of light containers, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The contractor charges €48 to load each heavy container and €36 to load each light container. How many of each should be loaded in order to maximise income?

(iii) On your graph, show the region where the income is at most €576.

SOLUTION

11 (a) (i)

$$m = \frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$$

$$\boxed{\begin{array}{cccc} (0, & 2) & (4, & 0) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_1 & y_1 & x_2 & y_2 \end{array}}$$

Slope: $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

Equation of a line: $\boxed{y - y_1 = m(x - x_1)}$

Equation of k: $y - 2 = -\frac{1}{2}(x - 0)$

$$y - 2 = -\tfrac{1}{2}x$$

$$2y - 4 = -x$$

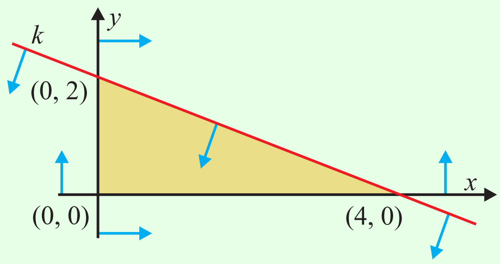
$$x + 2y - 4 = 0$$

11 (a) (ii)

Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side. The side of the line with (0, 0) obeys the inequality. The other side is the opposite to the inequality.

k: $x + 2y - 4 = 0$
Substitute (0, 0) into the equation k.

$(0)+2(0)-4$
$=0+0-4$
$=-4 \le 0$
$\Rightarrow x+2y-4 \le 0$



Other inequalities: $x \ge 0,\ y \ge 0$

11 (b) **Maximising and Minimising Problems**

Steps **1.** Choose two variables x and y to represent two different quantities. **2.** Draw up a table with restrictions and form the inequalities. **3.** Plot the lines in the same diagrams and shade the region satisfied by all the inequalities. **4.** Find the vertices of the region by solving the equations of the lines simultaneously. **5.** Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of heavy containers
Let y = Number of light containers

2.

	Heavy containers	Light containers	Restriction
Weight	$160x$	$40y$	2080
Time	$3x$	$2y$	54

Weight inequality: $160x+40y \le 2080 \Rightarrow 4x+y \le 52$

Time inequality: $3x+2y \le 54$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

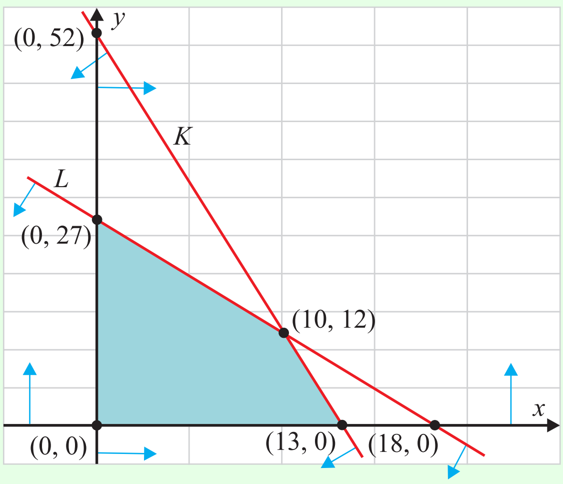
3. Plot the four inequalities.

Graph $4x+y \le 52$. Draw the line $4x + y = 52$. Call it K.

Intercepts: (0, 52), (13, 0). Test with (0, 0) $\Rightarrow 4(0)+(0)=0 \le 52$. This is true.
Shade the side of the line that contains (0, 0).

Graph $3x+2y \le 54$. Draw the line $3x + 2y = 54$. Call it L.

Intercepts: (0, 27), (18, 0). Test with (0, 0) $\Rightarrow 3(0)+2(0)=0 \le 54$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 27) and (13, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$4x + y = 52.....\mathbf{(1)}(\times -2)$$
$$3x + 2y = 54...\mathbf{(2)}$$

$$\begin{array}{l} -8x - 2y = -104 \\ \;\;3x + 2y = 54 \\ \hline -5x \qquad = -50 \Rightarrow x = 10 \end{array}$$

Substitute x = 10 back into Eqn. (**1**).

$\Rightarrow 4(10) + y = 52 \Rightarrow 40 + y = 52 \Rightarrow y = 12$

Therefore (10, 12) is the final vertex of the region.

5. Income = 48x + 36y is the function to be maximised.

	48x + 36y	**Income**
(0, 0)	48(0) + 36(0)	€0
(0, 27)	48(0) + 36(27)	€**972**
(10, 12)	48(10) + 36(12)	€912
(13, 0)	48(13) + 36(0)	€624

Therefore, 0 heavy containers and 27 light containers give the maximum income.

Answers

11 (b) (i) $4x + y \le 52,\ 3x + 2y \le 54$

(ii) $x = 0,\ y = 27$

11 (b) (iii)

$48x + 36y \le 576 \Rightarrow 4x + 3y \le 48$

Graph the straight line 4x + 3y = 48.
Intercepts: (0, 16), (12, 0).
Testing with (0, 0) gives a true statement which means you shade the side of the line containing (0, 0)