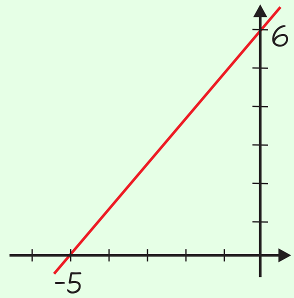
LINEAR PROGRAMMING (Q 11, PAPER 2)

2009

11 (a) The diagram shows the line $6x - 5y + 30 = 0$.



(i) Copy the diagram into your answer book and on it show the set of points which satisfy the inequality $6x - 5y + 30 \leq 0$.

(ii) Using the same diagram, illustrate the inequality $y \geq 2$.

(b) A person is setting up a new taxi firm. The firm will use medium-sized cars and large cars.
Each medium-sized car costs €20 000 and each large car costs €30 000.
The person has at most €300 000 to purchase the cars.

At any given time there are at most 13 drivers available to operate the taxis.

(i) Taking x as the number of medium-sized cars and y as the number of large cars, write down two inequalities in x and y and illustrate these inequalities on graph paper.

(ii) The estimate of the monthly profit on a medium-sized car is €800 and on a large car is €900. How many of each type of car should the person buy to maximise profit?

(iii) On your graph, show the region where the monthly profit is at most €7200.

SOLUTION

11 (a) (i)

> Take a test point like (0, 0) and substitute it into the inequality.
> If you get a true result, shade in the side of the line containing (0, 0).
> If you get a false result, shade in the side **not** containing (0, 0).

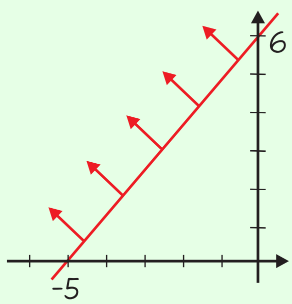
$6x - 5y + 30 \leq 0$

$6(0) - 5(0) + 30 \leq 0$

$0 + 0 + 30 \leq 0$

$30 \leq 0$

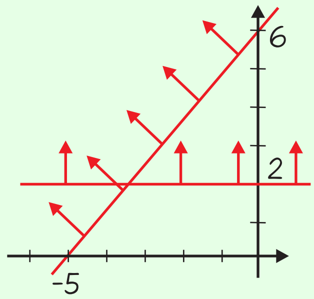
This is false so shade in the line on the opposite side to (0, 0).



11 (a) (ii)

$y \geq 2$ represents all the points above and including the line $y = 2$.

HORIZONTAL LINES:	$\geq$: ↑ (Above)
	$\leq$: ↓ (Below)



11 (b) MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of medium-sized cars
Let y = Number of large cars

2.

	Medium cars	Large cars	Restriction
Number	x	y	13
Cost	€20000x	€30000y	€300000

Numbers inequality: $x + y \leq 13$

Cost inequality: $20000x + 30000y \leq 300000 \Rightarrow 2x + 3y \leq 30$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

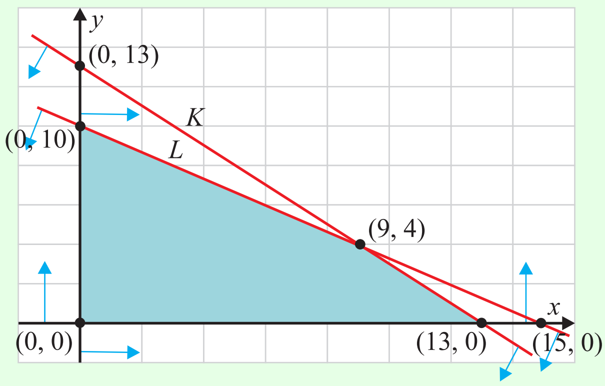
3. Plot the four inequalities.

Graph $x + y \leq 13$. Draw the line $x + y = 13$. Call it K.

Intercepts: (0, 13), (13, 0). Test with (0, 0) $\Rightarrow (0) + (0) = 0 \leq 13$. This is true. Shade the side of the line that contains (0, 0).

Graph $2x + 3y \leq 30$. Draw the line $2x + 3y = 30$. Call it L.

Intercepts: (0, 10), (15, 0). Test with (0, 0) $\Rightarrow 2(0) + 3(0) = 0 \leq 30$. This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 10) and (13, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$x + y = 13 \text{....}(\mathbf{1})\ (\times -2)$$
$$2x + 3y = 30 \text{..}(\mathbf{2})$$

$$-2x - 2y = -26$$
$$2x + 3y = 30$$
$$y = 4$$

Substitute y = 4 back into Eqn. (**1**).

$\Rightarrow x + (4) = 13 \Rightarrow x = 9$

Therefore (9, 4) is the final vertex of the region.

5. Profit = 800x + 900y is the function to be maximised.

	800x + 900y	**Profit**
(0, 0)	800(0) + 900(0)	€0
(0, 10)	800(0) + 900(10)	€9000
(9, 4)	800(9) + 900(4)	€**10800**
(13, 0)	800(13) + 900(0)	€10400

Therefore, 9 medium sized cars and 4 large cars give the maximum profit.

ANSWERS

11 (b) (i) $x + y \le 13,\ 2x + 3y \le 30$

(ii) x = 9, y = 4

11 (b) (iii)

$800x + 900y \le 7200 \Rightarrow 8x + 9y \le 72$

Graph the straight line 8x + 9y = 72.
Intercepts: (0, 8), (9, 0).
Testing with (0, 0) gives a true statement which means you shade the side of the line containing (0, 0)