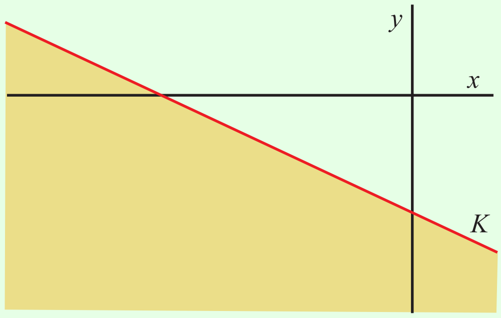
LINEAR PROGRAMMING (Q 11, PAPER 2)

2008

11 (a) (i) Does the point (18, –15) satisfy the inequality $3x+5y+11\geq 0$?
Justify your answer.

(ii) The equation of the line K is
$x + 2y + 4 = 0$.

Write down the inequality which defines the shaded half-plane in the diagram.



(b) A small restaurant offers two set lunch menus each day: a fish menu and a meat menu.
The fish menu costs €12 to prepare and the meat menu costs €18 to prepare.
The total preparation costs must not exceed €720.
The restaurant can cater for at most 50 people each lunchtime.

(i) Taking x as the number of fish menus ordered and y as the number of meat menus ordered, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The price of a fish menu is €25 and the price of a meat menu is €30. How many of each type would need to be ordered each day to maximise income?

(iii) Show that the maximum income does not give the maximum profit.

SOLUTION

11 (a) (i)

$3(18)+5(-15)+11$

$=54-65+11$

$=-10<0 \Rightarrow (18,\ -15)$ does not satisfy the inequality.

11 (a) (ii)

STEPS
1. Find the equation of the line. In this case it is given.
2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.
3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

1. $K: x+2y+4=0$

2. $(0)+2(0)+4=4>0$

3. $\therefore x+2y+4\leq 0$

11 (b) **Maximising and Minimising Problems**

Steps
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of fish menus
Let y = Number of meat menus

2.

	Fish menus	Meat menus	Restriction
Number	x	y	50
Cost	$12x$	$18y$	720

Numbers inequality: $x+y\le 50$

Cost inequality: $12x+18y\le 720 \Rightarrow 2x+3y\le 120$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

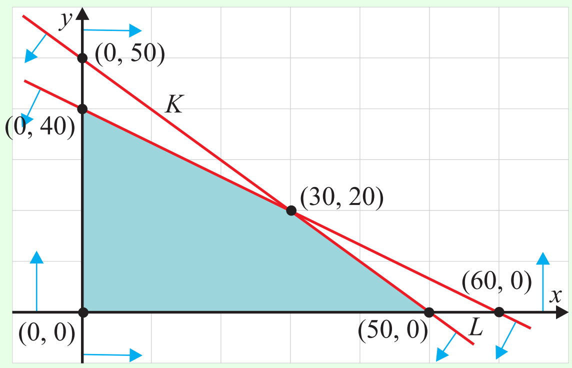
3. Plot the four inequalities.

Graph $x+y\le 50$. Draw the line $x + y = 50$. Call it K.

Intercepts: (0, 50), (50, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0\le 50$. This is true.
Shade the side of the line that contains (0, 0).

Graph $2x+3y\le 120$. Draw the line $2x + 3y = 120$. Call it L.

Intercepts: (0, 40), (60, 0). Test with (0, 0) $\Rightarrow 2(0)+3(0)=0\le 120$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 40) and (50, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$x + y = 50 \ldots\ldots(\mathbf{1}) \quad (\times -2)$$
$$2x + 3y = 120 \ldots(\mathbf{2})$$

$$-2x - 2y = -100$$
$$2x + 3y = 120$$
$$y = 20$$

Substitute y = 20 back into Eqn. (**1**).

$\Rightarrow x + (20) = 50 \Rightarrow x = 30$

Therefore (30, 20) is the final vertex of the region.

5. Income = 25x + 30y is the function to be maximised.

	25x + 30y	**Income**
(0, 0)	25(0) + 30(0)	€0
(0, 40)	25(0) + 30(40)	€1200
(30, 20)	25(30) + 30(20)	€**1350**
(50, 0)	25(50) + 30(0)	€1250

Therefore, 30 fish menus and 20 meat menus give the maximum income.

Profit = 13x + 12y is the function to be maximised.

	13x + 12y	**Income**
(0, 0)	13(0) + 12(0)	€0
(0, 40)	13(0) + 12(40)	€480
(30, 20)	13(30) + 12(20)	€630
(50, 0)	13(50) + 12(0)	€**650**

Therefore, 50 fish menus and 0 meat menus give the maximum profit.

ANSWERS

11 (b) (i) $x + y \le 50,\ 2x + 3y \le 120$

(ii) x = 30, y = 20