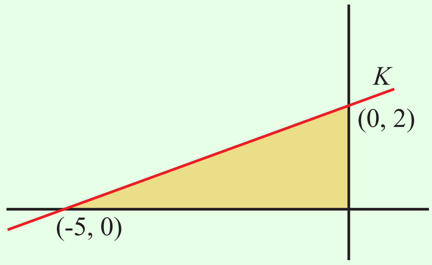
LINEAR PROGRAMMING (Q 11, PAPER 2)

2007

11 (a) The line K cuts the x-axis at $(-5, 0)$ and the y-axis at (0, 2).

(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.



(b) A developer is planning a holiday complex of cottages and apartments. Each cottage will accommodate 3 adults and 5 children and each apartment will accommodate 2 adults and 2 children. The other facilities in the complex are designed for a maximum of 60 adults and a maximum of 80 children.

(i) Taking x as the number of cottages and y as the number of apartments, write down two inequalities in x and y and illustrate these on graph paper.

(ii) If the rental income per night will be €65 for a cottage and €40 for an apartment, how many of each should the developer include in the complex to maximise potential rental income?

(iii) If the construction costs are €200 000 for a cottage and €120 000 for an apartment, how many of each should the developer include in the complex to minimise construction costs?

SOLUTION

11 (a) (i)

$$\begin{array}{cc} (-5, 0) & (0, 2) \\ \downarrow\ \downarrow & \downarrow\ \downarrow \\ x_1\ y_1 & x_2\ y_2 \end{array}$$

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ 3

Equation of a line: $y - y_1 = m(x - x_1)$ 4

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{0-(-5)} = \dfrac{2}{5}$

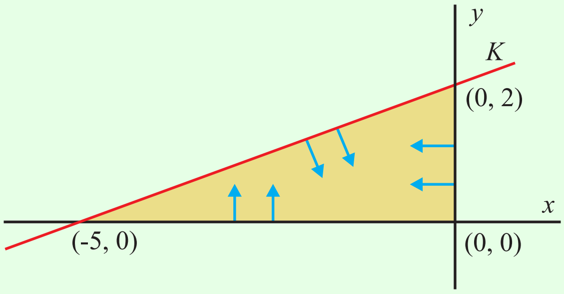
Equation of line K: $y - 0 = \frac{2}{5}(x - (-5)) \Rightarrow y = \frac{2}{5}(x+5)$

$\Rightarrow 5y = 2(x+5)$

$\Rightarrow 5y = 2x + 10$

$\therefore 2x - 5y + 10 = 0$

11 (a) (ii)



VERTICAL LINES: $\geq$: $\rightarrow$ (Right)
$\leq$: $\leftarrow$ (Left)

HORIZONTAL LINES: $\geq$: $\uparrow$ (Above)
$\leq$: $\downarrow$ (Below)

Inequality 1: Above the *x*-axis $\Rightarrow x \geq 0$

Inequality 2: Left of the *y*-axis $\Rightarrow y \leq 0$

STEPS

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow 2(0)-5(0)+10=10 \geq 0$

3. The indicated region is on the same side as (0, 0).

Therefore, $2x-5y+10 \geq 0$ is the inequality of the indicated region.

Three inequalities: $x \leq 0,\ y \geq 0,\ 2x-5y+10 \geq 0$

11 (b) MAXIMISING AND MINIMISING PROBLEMS

STEPS

1. Choose two variables *x* and *y* to represent two different quantities.

2. Draw up a table with restrictions and form the inequalities.

3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.

4. Find the vertices of the region by solving the equations of the lines simultaneously.

5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of cottages
Let y = Number of apartments

2.

	Cottages	Apartments	Restriction
Adults	$3x$	$2y$	60
Children	$5x$	$2y$	80

Adults inequality: $3x+2y \leq 60$

Children inequality: $5x+2y \leq 80$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

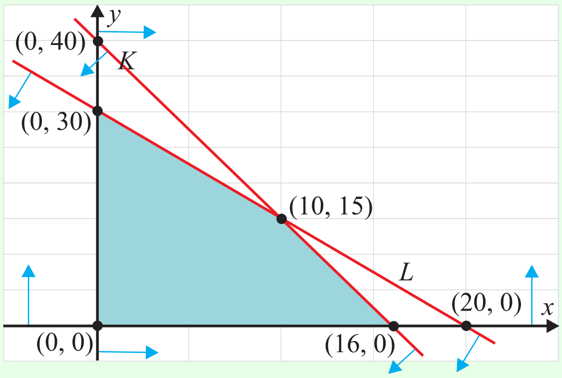
3. Plot the four inequalities.

Graph $3x+2y\leq 60$. Draw the line $3x+2y=60$. Call it *K*.

Intercepts: (0, 30), (20, 0). Test with (0, 0) $\Rightarrow 3(0)+2(0)=0\leq 60$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x+2y\leq 80$. Draw the line $5x+2y=80$. Call it *L*.

Intercepts: (0, 40), (16, 0). Test with (0, 0) $\Rightarrow 5(0)+2(0)=0\leq 80$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (16, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$3x+2y=60 \ldots \mathbf{(1)}$$
$$5x+2y=80 \ldots \mathbf{(2)} \quad (\times -1)$$

$$\begin{aligned} 3x+2y &= 60 \\ -5x-2y &= -80 \\ \hline -2x \quad\quad &= -20 \Rightarrow x=10 \end{aligned}$$

Substitute *x* = 10 back into Eqn. (**1**).
$\Rightarrow 3(10)+2y=60\Rightarrow 2y=30\Rightarrow y=15$
Therefore (10, 15) is the final vertex of the region.

5. Rental income = 65*x* + 40*y* is the function to be minimised.

	65*x* + 40*y*	**Income**
(0, 0)	65(0) + 40(0)	€0
(0, 30)	65(0) + 40(30)	€1200
(10, 15)	65(10) + 40(15)	€**1250**
(16, 0)	65(16) + 40(0)	€1040

Therefore, 10 cottages and 15 apartments give the maximum rental income.

Construction costs = 200000*x* + 120000*y* is the function to be minimised.

	200000*x* + 120000*y*	**Cost**
(0, 0)	200000(0) + 120000(0)	€0
(0, 30)	200000(0) + 120000(30)	€3,600,000
(10, 15)	200000(10) + 120000(15)	€3,800,000
(16, 0)	200000(16) + 120000(0)	€**3,200,000**

Therefore, 16 cottages and 0 apartments give the minimum construction costs.

Answers

11 (b) **(i)** $3x + 2y \le 60,\ 5x + 2y \le 80$

(ii) 10 cottages, 15 apartments

(iii) 16 cottages and 0 apartments