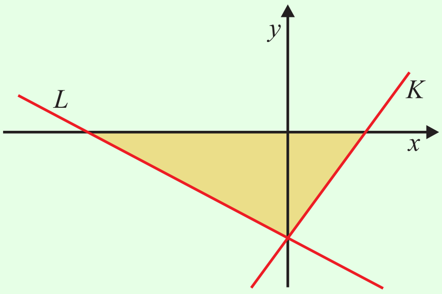
LINEAR PROGRAMMING (Q 11, PAPER 2)

2006

11 (a) The equation of the line L is $5x + 8y + 40 = 0$.
The equation of the line K is $10x - 7y - 35 = 0$.
Write down the 3 inequalities that together define the shaded region in the diagram.



(b) Due to a transport disruption, a bus company is contracted at short notice to carry up to 1500 passengers to complete their journey. Passengers not carried by this company will be carried by a taxi company.

The bus company has available standard buses and mini-buses. Each standard bus carries 60 passengers and each mini-bus carries 30 passengers.

Each bus is operated by one driver and there are at most 30 drivers available.

(i) Taking x as the number of standard buses and y as the number of mini-buses, write down two inequalities in x and y and illustrate them on graph paper.

(ii) The operating profit for the journey is €80 for a standard bus and €50 for a minibus. How many of each type of bus should be used in order to maximise the profit?

(iii) If the bus company paid each driver a bonus for working at short notice, the operating profit for each bus would be reduced by €30. By how much would this decrease the maximum profit available to the company?

SOLUTION

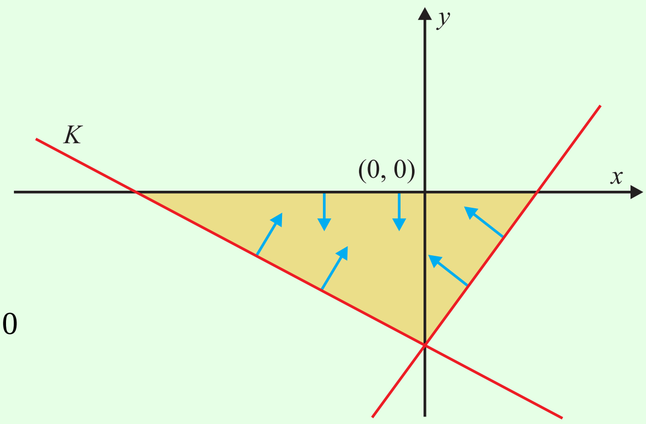
11 (a)

Equation of L: $5x + 8y + 40 = 0$

Test with (0, 0): $5(0)+8(0)+40=40 \geq 0$

Shaded side is on the same side as (0, 0).

$\therefore 5x+8y+40 \geq 0$



Equation of K: $10x-7y-35=0$

Test with (0, 0): $10(0)-7(0)-35=-35 \leq 0$

Shaded side is on the same side as (0, 0).

$\therefore 10x-7y-35 \leq 0$

Below the x-axis: $y = 0$
Shaded side under this line.
$\therefore y \leq 0$

HORIZONTAL LINES: $\geq$: ↑ (Above)
$\leq$: ↓ (Below)

11 (b)

MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of standard buses
Let y = Number of mini-buses

2.

	Standard buses	Mini-buses	Restriction
Passengers	$60x$	$30y$	1500
Drivers	x	y	30

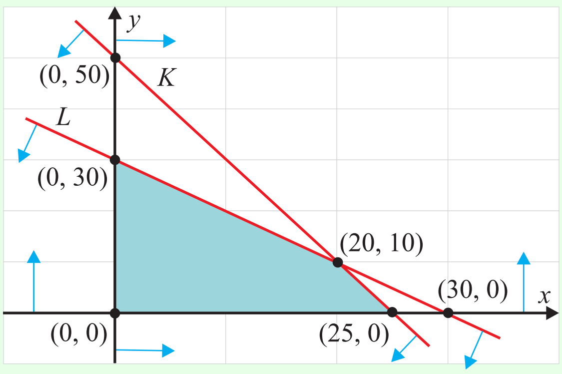
Passsengers inequality: $60x+30y \le 1500 \Rightarrow 2x+y \le 50$

Drivers inequality: $x+y \le 30$

As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.
Graph $2x+y \le 50$. Draw the line $2x + y = 50$. Call it K.
Intercepts: (0, 50), (25, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0 \le 50.$ This is true.
Shade the side of the line that contains (0, 0).

Graph $x+y \le 30$. Draw the line $x + y = 30$. Call it L.
Intercepts: (0, 30), (30, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0 \le 30.$
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 30) and (25, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x + y = 50 \ldots(\mathbf{1})$$
$$x + y = 30 \ldots(\mathbf{2}) \quad (\times -1)$$

$$2x + y = 50$$
$$-x - y = -30$$
$$x \qquad = 20$$

Substitute x = 20 back into Eqn. (**2**).

$\Rightarrow (20) + y = 30 \Rightarrow y = 10$

Therefore (20, 10) is the final vertex of the region.

5. Profit = 80x + 50y is the function to be maximised.

	$80x + 50y$	**Profit**
(0, 0)	80(0) + 50(0)	€0
(0, 30)	80(0) + 50(30)	€1500
(20, 10)	80(20) + 50(10)	€**2100**
(25, 0)	80(25) + 50(0)	€2000

Therefore, 20 standard buses and 10 mini-buses give the maximum profit.

ANSWERS

11 (b) (i) $x + y \leq 30,\ 2x + y \leq 50$

11 (b) (ii) $x = 20,\ y = 10$

11 (b) (iii)

Profit = 50x + 20y is the function to be maximised now that the operating profit for each bus is reduced by €30.

	$50x + 20y$	**Profit**
(0, 0)	50(0) + 20(0)	€0
(0, 30)	50(0) + 20(30)	€600
(20, 10)	50(20) + 20(10)	€1200
(25, 0)	50(25) + 20(0)	€**1250**

The maximum profit would now be for 25 standard buses and 0 mini-buses.
Therefore, the decrease in profit would €**2100** – €**1250** = €**850**