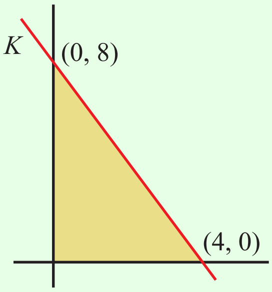
LINEAR PROGRAMMING (Q 11, PAPER 2)

2005

11 (a) The line K cuts the x-axis at (4, 0) and the y-axis at (0, 8).



(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.

(b) A manufacturer of garden furniture produces plastic chairs and tables. Each chair requires 2 kg of raw material and each table requires 5 kg of raw material. In any working period the raw material used cannot exceed 800 kg.

Each chair requires 4 minutes of machine time and each table requires 4 minutes of machine time. The total machine time available in any working period is 1000 minutes.

(i) Taking x as the number of chairs and y as the number of tables, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The manufacturer sells each chair for €20 and each table for €40. How many of each should be produced in each working period to maximise income?

(iii) The manufacturer's costs for each chair are €17 and for each table are €34.70. Express the profit as a percentage of income, assuming the income has been maximised.

SOLUTION

11 (a) (i)

$$\begin{matrix}(0, & 8) & (4, & 0)\\ \downarrow & \downarrow & \downarrow & \downarrow\\ x_1 & y_1 & x_2 & y_2\end{matrix}$$

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **3**

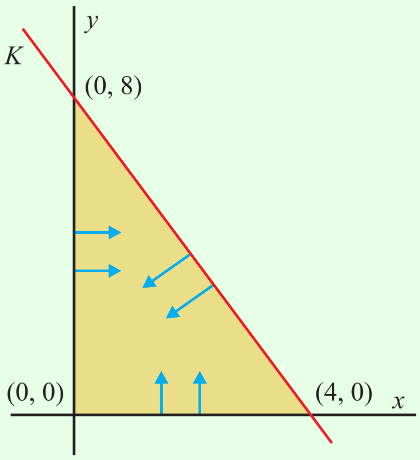
Equation of a line: $y - y_1 = m(x - x_1)$ **4**

Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-8}{4-0} = \dfrac{-8}{4} = -2$

Equation of line K: $y - 8 = -2(x - 0) \Rightarrow y - 8 = -2x$

$\therefore 2x + y - 6 = 0$

11 (a) (ii)



VERTICAL LINES:	≥: → (Right) ≤: ← (Left)

HORIZONTAL LINES:	≥: ↑ (Above) ≤: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y \geq 0$

STEPS

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow 2(0)+(0)-8=-8 \leq 0$

3. The indicated region is on the same side as (0, 0).

Therefore, $2x+y-8 \leq 0$ is the inequality of the indicated region.

Three inequalities: $x \geq 0,\ y \geq 0,\ 2x+y-8 \leq 0$

11 (b) MAXIMISING AND MINIMISING PROBLEMS

STEPS

1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of chairs

Let y = Number of tables

2.

	Chairs	Tables	Restriction
Raw material	$2x$	$5y$	800
Time	$4x$	$4y$	1000

Raw material inequality: $2x+5y \leq 800$

Time inequality: $4x+4y \leq 1000 \Rightarrow x+y \leq 250$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

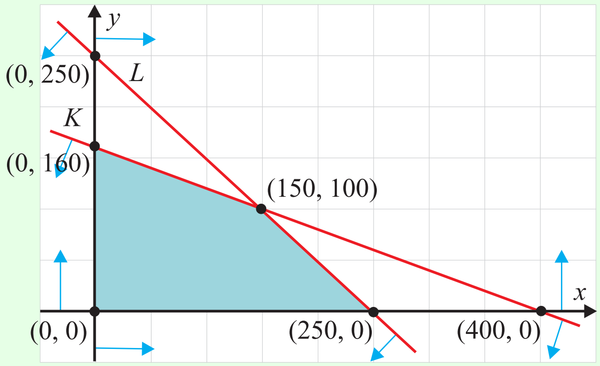
3. Plot the four inequalities.

Graph $2x+5y \le 800$. Draw the line $2x + 5y = 800$. Call it K.

Intercepts: (0, 160), (400, 0). Test with (0, 0) $\Rightarrow 2(0)+5(0)=0 \le 800$. This is true. Shade the side of the line that contains (0, 0).

Graph $x+y \le 250$. Draw the line $x + y = 250$. Call it L.

Intercepts: (0, 250), (250, 0). Test with (0, 0) $\Rightarrow (0)+(0)=0 \le 250$. This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 160) and (250, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$2x+5y=800 \ldots \mathbf{(1)}$$
$$x+y=250 \ldots \mathbf{(2)} \quad (\times -2)$$

$$\begin{aligned} 2x+5y &= 800 \\ -2x-2y &= -500 \\ \hline 3y &= 300 \Rightarrow y=100 \end{aligned}$$

Substitute $y = 100$ back into Eqn. (**2**).

$\Rightarrow x+(100)=250 \Rightarrow x=250-100=150$

Therefore (150, 100) is the final vertex of the region.

5. Income = $20x + 40y$ is the function to be minimised.

	$20x + 40y$	**Income**
(0, 0)	20(0) + 40(0)	€0
(0, 160)	20(0) + 40(160)	€6400
(150, 100)	20(150) + 40(100)	**€7000**
(250, 0)	20(250) + 40(0)	€5000

Therefore, 150 chairs and 100 tables give the maximum income.

11 (b) (i) $2x+5y \le 800,\ x+y \le 250$

11 (b) (ii) $x = 150$, $y = 100$

11 (b) (iii)

Manufacturing costs $= 150 \times €17 + 100 \times €34.70 = €6020$

Profit $= €7000 - €6020 = €980$

Profit as a percentage of income $= \dfrac{€980}{€7000} \times 100\% = 14\%$