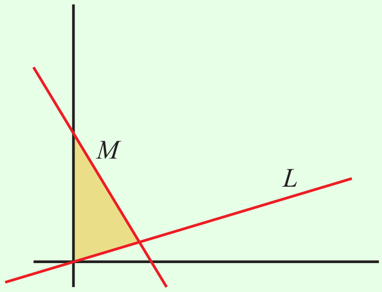
LINEAR PROGRAMMING (Q 11, PAPER 2)

2004

11 (a) The equation of the line L is $x-2y=0$.
The equation of the line M is $2x+y=4$.
Write down the three inequalities that together define the shaded region in the diagram.



(b) A shop-owner displays videos and DVDs in his shop.
Each video requires 720 cm^3 of display space and each DVD requires 360 cm^3 of display space. The available display space cannot exceed 108 000 cm^3. The shopowner buys each video for €6 and each DVD for €8. He does not wish to spend more than €1200.

(i) Taking x as the number of videos and y as the number of DVDs, write down two inequalities in x and y and illustrate these on graph paper.

During a DVD promotion the selling price of a video is €11 and of a DVD is €10. Assuming that the shop-owner can sell all the videos and DVDs,

(ii) how many of each type should he display in order to maximise his income?

(iii) how many of each type should he display in order to maximise his profit?

SOLUTION

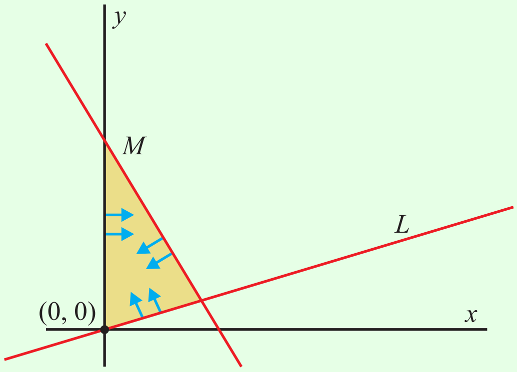
11 (a)

Equation of L: $x-2y=0$

Test with (0, 1): $(0)-2(1)=-2\le 0$

Shaded side is on the same side as (0, 1).

$\therefore x-2y\le 0$



Equation of M: $2x+y=4$

Test with (0, 0): $2(0)+(0)=0\le 4$

Shaded side is on the same side as (0, 0).

$\therefore 2x+y\le 4$

Right of the y-axis: $x=0$
Shaded side under this line.

$\therefore x\ge 0$

VERTICAL LINES:	$\ge$: $\rightarrow$ (Right) $\le$: $\leftarrow$ (Left)

ANSWER: $x-2y\le 0,\ x\ge 0,\ 2x+y\le 4$

11 (b) **MAXIMISING AND MINIMISING PROBLEMS**

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of videos
Let y = Number of DVDs

2.

	Videos	DVDs	Restriction
Space	$720x$	$360y$	108,000
Cost	$6x$	$8y$	1200

Space inequality: $720x+360y\le 108000 \Rightarrow 2x+y\le 300$

Cost inequality: $6x+8y\le 1200 \Rightarrow 3x+4y\le 600$

As always, there are two inequalities that are obvious: $x\ge 0$ and $y\ge 0$.

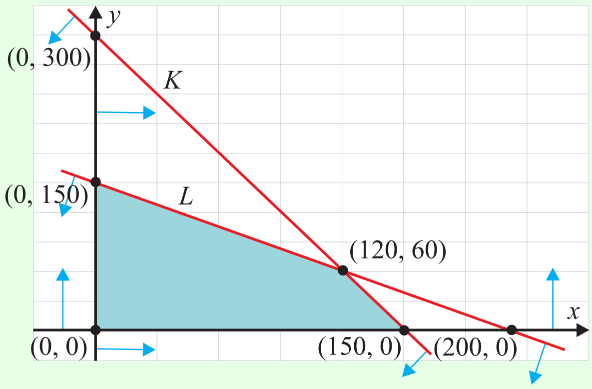
3. Plot the four inequalities.

Graph $2x+y\le 300$. Draw the line $2x+y=300$. Call it K.

Intercepts: (0, 300), (150, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0\le 300$. This is true.
Shade the side of the line that contains (0, 0).

Graph $3x+4y\le 600$. Draw the line $3x+4y=600$. Call it L.

Intercepts: (0, 150), (200, 0). Test with (0, 0) $\Rightarrow 3(0)+4(0)=0\le 600$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 150) and (150, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$2x + y = 300 \ldots(\mathbf{1}) \quad (\times -4)$$
$$3x + 4y = 600 \ldots(\mathbf{2})$$

$$-8x - 4y = -1200$$
$$3x + 4y = 600$$
$$-5x = -600 \Rightarrow x = 120$$

Substitute *x* = 120 back into Eqn. (**1**).

$\Rightarrow 2(120) + y = 300 \Rightarrow y = 300 - 240 = 60$

Therefore (120, 60) is the final vertex of the region.

5. Income = 11*x* + 10*y* is the function to be maximised.

	11*x* + 10*y*	**Income**
(0, 0)	11(0) + 10(0)	€0
(0, 150)	11(0) + 10(150)	€1500
(120, 60)	11(120) + 10(60)	€**1920**
(150, 0)	11(150) + 10(0)	€1650

Therefore, 120 videos and 60 DVDs give the maximum income.

ANSWERS

11 (b) (i) $2x + y \le 300,\ 3x + 4y \le 600$

11 (b) (ii) *x* = 120, *y* = 60

11 (b) (iii)

Profit on videos = €11 – €6 = €5
Profit on DVDs = €10 – €8 = €2

Profit = 5*x* + 3*y* is the function to be maximised.

	5*x* + 2*y*	**Income**
(0, 0)	5(0) + 2(0)	€0
(0, 150)	5(0) + 2(150)	€300
(120, 60)	5(120) + 2(60)	€720
(150, 0)	5(150) + 2(0)	€**750**

Therefore, 150 videos and 0 DVDs will maximise profit.