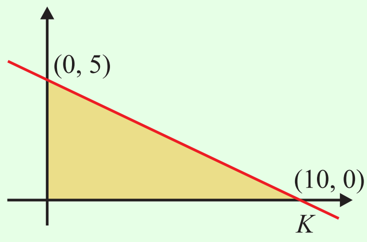
LINEAR PROGRAMMING (Q 11, PAPER 2)

2003

11 (a) The line K cuts the x-axis at (10, 0) and the y-axis at (0, 5).



(i) Find the equation of K.

(ii) Write down the three inequalities that together define the region enclosed by K, the x-axis and the y-axis.

(b) A developer is planning a scheme of holiday homes, consisting of large and small bungalows. Each large bungalow will accommodate 8 people and each small bungalow will accommodate 6 people. The development is not permitted to accommodate more than 216 people. The floor area of each large bungalow is 200 m^2 and the floor area of each small bungalow is 100 m^2. The total floor area of all the bungalows must not exceed 4000 m^2.

(i) Taking x as the number of large bungalows and y as the number of small bungalows, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The expected net annual income from each large bungalow is €14 000 and from each small bungalow is €8000. How many of each type should be built in order to maximise the total expected net annual income?

(iii) The developer decides to build as indicated in part **(ii)**. The cost of building each large bungalow is €110 000 and the cost of building each small bungalow is €85 000. The total cost of the development is equal to the building costs plus €1.58 million. How many years will it take to recoup the total cost of the development?

SOLUTION

11 (a) (i)

(0, 5)	(10, 0)
↓ ↓	↓ ↓
x_1 y_1	x_2 y_2

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **3**

Equation of a line: $y - y_1 = m(x - x_1)$ **4**

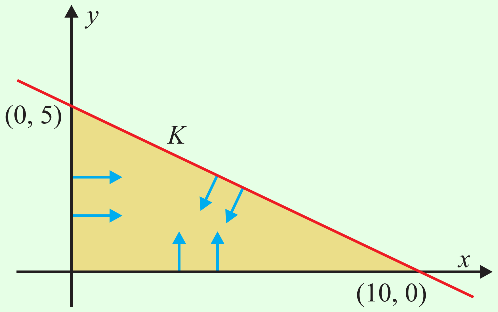
Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-5}{10-0} = \dfrac{-5}{10} = -\dfrac{1}{2}$

Equation of line K: $y - 5 = -\frac{1}{2}(x - 0) \Rightarrow 2(y-5) = -x$

$\Rightarrow 2y - 10 = -x$

$\therefore x + 2y - 10 = 0$

11 (a) (ii)



Vertical Lines: ≥: → (Right) ≤: ← (Left)

Horizontal Lines: ≥: ↑ (Above) ≤: ↓ (Below)

Inequality 1: Above the x-axis $\Rightarrow x \geq 0$

Inequality 2: Right of the y-axis $\Rightarrow y \geq 0$

Steps

2. Substitute a test point (usually (0, 0)) into the equation of the line. The left-hand side will be either less than or greater than the right-hand side.

3. The side of the line with (0, 0) obeys the inequality found in Step **2**. The other side is the opposite to the inequality found in Step **2**.

Inequality 3:

2. Substitute (0, 0) into $K \Rightarrow (0)+2(0)-10=-10 \leq 0$

3. The indicated region is on the same side as (0, 0).

Therefore, $x+2y-10 \leq 0$ is the inequality of the indicated region.

Three inequalities: $x+2y-10 \leq 0,\ y \geq 0,\ x \geq 0$

11 (b) **Maximising and Minimising Problems**

Steps

1. Choose two variables x and y to represent two different quantities.

2. Draw up a table with restrictions and form the inequalities.

3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.

4. Find the vertices of the region by solving the equations of the lines simultaneously.

5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of large bungalows
Let y = Number of small bungalows

2.

	Large bungalows	Small bungalows	Restriction
Accommodation	$8x$	$6y$	216
Floor Space	$200x$	$100y$	4000

Accommodation inequality: $8x+6y \leq 216 \Rightarrow 4x+3y \leq 108$

Floor space inequality: $200x+100y \leq 4000 \Rightarrow 2x+y \leq 40$

As always, there are two inequalities that are obvious: $x \geq 0$ and $y \geq 0$.

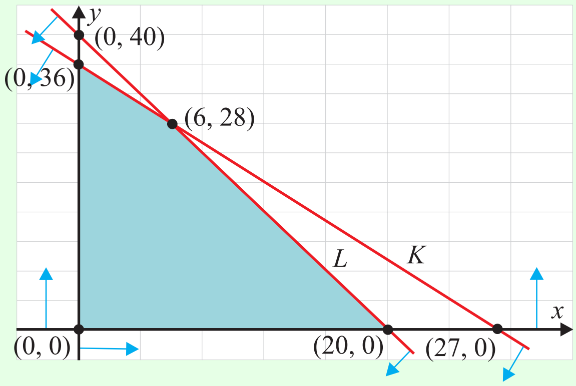
3. Plot the four inequalities.

Graph $4x+3y \leq 108$. Draw the line $4x + 3y = 108$. Call it K.

Intercepts: (0, 36), (27, 0). Test with (0, 0) $\Rightarrow 4(0)+3(0)=0 \leq 108$. This is true.
Shade the side of the line that contains (0, 0).

Graph $2x+y \leq 40$. Draw the line $2x + y = 40$. Call it L.

Intercepts: (0, 40), (20, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0 \leq 40$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 36) and (20, 0).
The only one you need to work out simultaneously is where the lines K and L intersect.

$$4x+3y=108 \ldots(\mathbf{1})$$
$$2x+y\ =40 \ldots\ldots(\mathbf{2})\ \ (\times -3)$$

$$\begin{aligned} 4x+3y &= 108 \\ -6x-3y &= -120 \\ \hline -2x &= -12 \Rightarrow x=6 \end{aligned}$$

Substitute $x = 6$ back into Eqn. (**2**).

$\Rightarrow 2(6)+y=40 \Rightarrow y=40-12=28$

Therefore (6, 28) is the final vertex of the region.

5. Income = $14000x + 8000y$ is the function to be maximised.

	$14000x + 8000y$	**Income**
(0, 0)	14000(0) + 8000(0)	€0
(0, 36)	14000(0) + 8000(36)	€288,000
(6, 28)	14000(6) + 8000(28)	€**308,000**
(20, 0)	14000(20) + 8000(0)	€280,000

Therefore, 6 large bungalows and 28 small bungalows give the maximum rental income.

Answers

11 (b) (i) $4x+3y \leq 108,\ 2x+y \leq 40$

11 (b) (ii) $x = 6, y = 28$

11 (b) (iii)

Cost of development $= 6 \times €110000 + 28 \times €85000 + €1580000 = €4,620,000$

No. of years $= \dfrac{€4,620,000}{€308,000} = 15$