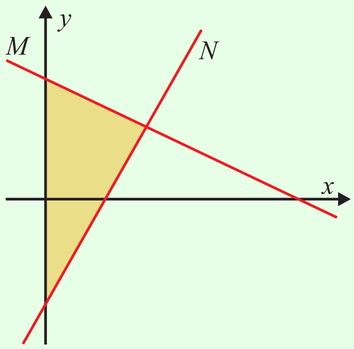
LINEAR PROGRAMMING (Q 11, PAPER 2)

2002

11 (a) The equation of the line M is $2x + y = 10$.
The equation of the line N is $4x - y = 8$.

Write down the three inequalities that define the shaded region in the diagram.



(b) A new ship is being designed. It can have two types of cabin accommodation for passengers — type A cabins and type B cabins.

Each type A cabin accommodates 6 passsengers and each type B cabin accommodates 3 passengers. The maximum number of passengers that the ship can accommodate is 330.

Each type A cabin occupies 50 m^3 of floor space. Each type B cabin occupies 10 m^3 of floor space. The total amount of floor space occupied by cabins cannot exceed 2300 m^3.

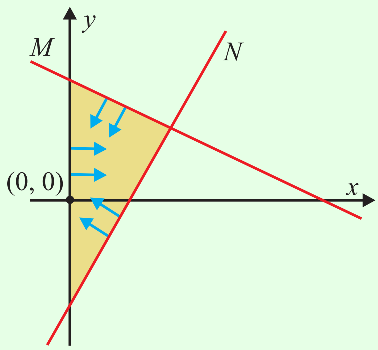
(i) Taking x to represent the number of type A cabins and y to represent the number of type B cabins, write down two inequalities in x and y and illustrate these on graph paper.

(ii) The income on each voyage from renting the cabins to passengers is €600 for each type A cabin and €180 for each type B cabin. How many of each type of cabin should the ship have so as to maximise income, assuming that all cabins are rented?

(iii) What is the maximum possible income on each voyage from renting the cabins?

SOLUTION

11 (a)



Equation of M: $2x + y = 10$

Test with (0, 0): $2(0)+(0)=0 \le 10$

Shaded side is on the same side as (0, 0).

$\therefore 2x + y \le 10$

Equation of N: $4x - y = 8$

Test with (0, 0): $4(0)-(0)=0 \le 8$

Shaded side is on the same side as (0, 0).

$\therefore 4x - y \le 8$

y-axis: $x = 0$

Shaded side is right of this line.

$\therefore x \ge 0$

VERTICAL LINES:	$\ge$: $\rightarrow$ (Right) $\le$: $\leftarrow$ (Left)

11 (b) MAXIMISING AND MINIMISING PROBLEMS

STEPS
1. Choose two variables x and y to represent two different quantities.
2. Draw up a table with restrictions and form the inequalities.
3. Plot the lines in the same diagrams and shade the region satisfied by all the inequalities.
4. Find the vertices of the region by solving the equations of the lines simultaneously.
5. Maximise or minimise the given functions by substituting the coordinates of the vertices into the function.

1. Let x = Number of Type A cabins
Let y = Number of Type B cabins

2.

	Type A	Type B	Restriction
Accommodation	$6x$	$3y$	330
Floor space	$50x$	$10y$	2300

Accommodation inequality: $6x+3y \le 330 \Rightarrow 2x+y \le 110$

Floor space inequality: $50x+10y \le 2300 \Rightarrow 5x+y \le 230$

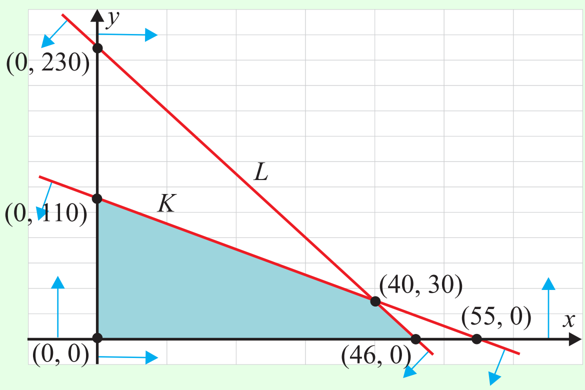
As always, there are two inequalities that are obvious: $x \ge 0$ and $y \ge 0$.

3. Plot the four inequalities.
Graph $2x+y \le 110$. Draw the line $2x + y = 110$. Call it K.

Intercepts: (0, 110), (55, 0). Test with (0, 0) $\Rightarrow 2(0)+(0)=0 \le 110$. This is true.
Shade the side of the line that contains (0, 0).

Graph $5x+y \le 230$. Draw the line $5x + y = 230$. Call it L.

Intercepts: (0, 230), (46, 0). Test with (0, 0) $\Rightarrow 5(0)+(0)=0 \le 230$.
This is true. Shade the side of the line that contains (0, 0).



4. You already know the coordinates of the vertices of the shaded region that are on the axes: (0, 0), (0, 110) and (46, 0).
The only one you need to work out simultaneously is where the lines *K* and *L* intersect.

$$2x + y = 110 ...(\mathbf{1}) \quad (\times -1)$$
$$5x + y = 230 ...(\mathbf{2})$$

$$\begin{array}{rl} -2x - y &= -110 \\ 5x + y &= 230 \\ \hline 3x &= 120 \Rightarrow x = 40 \end{array}$$

Substitute x = 40 back into Eqn. (**1**).

$$\Rightarrow 2(40) + y = 110 \Rightarrow y = 110 - 80 = 30$$

Therefore (40, 30) is the final vertex of the region.

5. Income = 600x + 180y is the function to be maximised.

	600x + 180y	**Income**
(0, 0)	600(0) + 180(0)	€0
(0, 110)	600(0) + 180(110)	€19,800
(40, 30)	600(40) + 180(30)	€**29,400**
(46, 0)	600(46) + 180(0)	€27,600

Therefore, 40 type A cabins and 30 type B cabins give the maximum rental income.

Answers

11 (b) (i) $2x + y \leq 110,\ 5x + y \leq 230$

11 (b) (ii) $x = 40,\ y = 30$

11 (b) (iii) €29,400